# ANALYTICAL INVESTIGATION OF TRANSIENT DIFFUSER WAVE PHENOMENA

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# NAVAL POSTGRADUATE SCHOOL
## Monterey, California

# THESIS

ANALYTICAL INVESTIGATION OF TRANSIENT
DIFFUSER WAVE PHENOMENA

by

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Analytical Investigation of Transient

Diffuser Wave Phenomena

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| | |
|---|---|
| $a$ | speed of sound |
| $A$ | area |
| $c_p$ | specific heat at constant pressure |
| $c_v$ | specific heat at constant volume |
| $\frac{D()}{Dt}$ | derivative in the direction of a particle path (substantial derivative) |
| $e$ | gas internal energy per unit mass |
| $f$ | frictional dissipative forces per unit mass |
| HPS | area effect term $(\frac{\gamma-1}{2})\frac{ua}{A}\frac{dA}{dx}$ integrated from points S to P |
| HPQ | area effect term $(\frac{\gamma-1}{2})\frac{ua}{A}\frac{dA}{dx}$ integrated from points S to P |
| $L_0$ | duct length |
| $p$ | pressure |
| $P$ | $p^{\frac{\gamma-1}{2\gamma}}$ (computational variable) |
| $R$ | gas constant |
| $s$ | specific entropy |
| $s_*$ | $s_0+2c_p \ln a_0-R \ln p_0$ |
| $t$ | time (nondimensionalized as $\frac{a_0 t}{L_0}$) |
| $u$ | velocity |
| $U$ | $(\frac{\gamma-1}{2})u$ (computational variable) |
| $x$ | distance, duct position (nondimensionalized with respect to duct length) |
| $\rho$ | density |
| $\gamma$ | $c_p/c_v$ ratio of specific heats |
| $\sigma$ | $\exp[(s-s_*)/2c_p)]$ (computational variable) |

Subscripts

$_0$ reference conditions (ambient conditions utilized throughout)

P,S,Q,R values at points P,S,Q,R in the x-t plane

# I. INTRODUCTION

Gas Dynamic Lasers (GDLs) typically operate at cavity Mach numbers of 4.0 to 5.5 in order to obtain the kinetics desired for population inversion and signal gain. GDL operation often consist of a series of short bursts which result in periods of supersonic flow followed by periods of no flow. Although efficient pressure recovery is not of prime importance, the high Mach number, low pressure flow must be diffused in order to permit exhausting to the atmosphere. While rapid and efficient diffuser start-up is required in all laser applications due to its effect on lasing performance, it becomes especially critical in the short burst mode of operation.

Some chemical lasers also require diffusing of low pressure high flow rate gases. Since these gases are collected and condensed or absorbed, diffuser efficiencies become more important in this type of application.

Aircraft installation of lasers compounds the above problems with size, weight and pressure ratio limitations. Diffusers used on GDLs currently constitute a large percentage of the laser size and hence become prohibitive in airborne installations. If an aircraft axial flow compressor is used for gas supply, pressure ratios are limited to approximately 30 to 1.

The diffuser starting problem is similar to that obtained in starting supersonic wind tunnels. Extensive work was done

in this area following World War II and into the early 1950's. Some of the means which have been utilized in starting these tunnels are variable area diffusers, overpressure, perforated walls, auxiliary air injection, and boundary layer control [Ref. 6].

In conjunction with work being done for NAVAIR at China Lake on the airborne gas dynamic laser, research is being undertaken at the Naval Postgraduate School on minimizing diffuser size requirements and starting time. The research has proceeded along two directions; experimental and analytical. The experimental investigation of diffuser start-up has been conducted by LCDR J. Zerr [Ref. 8], utilizing a blow-down wind tunnel to establish Mach 3 and Mach 4 flow. As a first step in the analytical program it was desired to have a computer model by which various flow conditions could be analyzed.

A complete analysis of the starting process would require a four-dimensional approach: 3 spatial coordinates plus the time coordinate. In such an approach, discontinuities and non-linearities caused by the presence of shock waves create problems of enormous complexity.

One can obtain insight into the starting phenomenon through a quasi-one-dimensional analysis which utilizes one space and one time coordinate. Transverse area-change effects may be included if the radius of curvature of the change is large in comparison with the axial duct length.

Rudinger [Ref. 5] treats the one-dimensional unsteady, non-linear wave problem for flow in ducts. The procedures he develops allow for the inclusion of heat addition, variable entropy, variable area and several boundary conditions and discontinuities. Since his method is designed for hand graphical analysis, however, it does not lend itself directly to computer numerical methods.

Spalding [Ref. 7] has modified the techniques of Rudinger in order to permit efficient digital computation procedures. His modifications consist primarily of the implementation of a fixed spatial grid combined with a backward interpolation of wave characteristics. Additionally, the Riemann variables for the characteristics have been re-defined for ease of computation.

Solution of the characteristic-wave problem is also discussed extensively in Refs. 1, 3, and 6. Several alternatives are offered for handling the iteration process which inevitably results from the inclusion of the area effect term. The complexity of these schemes, in any case, must be weighed against the increase in accuracy and the obvious limitations of the one-dimensional analysis.

## II. APPROACH

The computer program developed in this thesis is based on the quasi-one-dimensional unsteady wave analysis described in Ref. 5 as modified by Ref. 7. These techniques have been utilized in the development of a previous computer program, PREX, for use with cyclic, constant area pressure exchangers. Since many of the subroutines used in PREX were general in nature, they were adapted as a starting point for this thesis and extensions were developed to permit the calculation of supersonic flow through ducts of varying area.

The program essentially solves a shock-tube type problem in a duct with one closed end and one open end, (i.e., a Ludweig tube). The nozzle and diffuser are located between the diaphragm and the open end. When the diaphragm is broken, pressure waves travel through the nozzle/diffuser section while rarefaction waves proceed toward the closed end. Since the diaphragm is located closer to the open end, the diffuser starts prior to the time the rarefaction waves reach the closed end. In this manner the diffuser start-up process is isolated from the reflected rarefaction waves returning from the closed end in order to more closely approximate a reservoir boundary condition. The computational model is depicted in Figure 1.

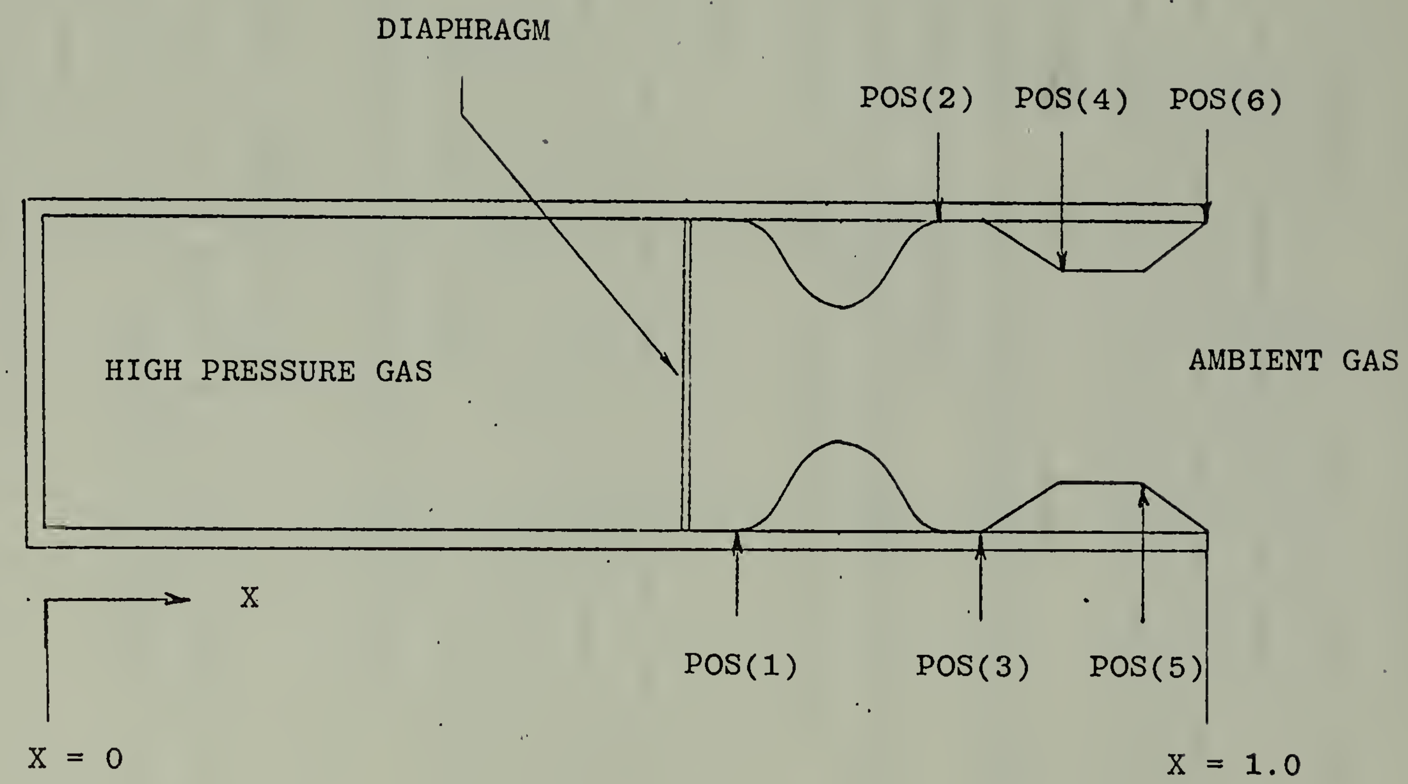


Figure 1. Duct Geometry.

# III. DERIVATION OF EQUATIONS

In the derivations of the characteristic equations, the following implicit assumptions have been made:

1. The gas flow obeys the ideal gas relations.
2. The gas is calorically perfect (constant specific heats).
3. The flow is considered quasi-one-dimensional (area effects are included).
4. The duct is rigid in time with no mass flux through the walls (e.g., no blowing or suction).

## A. CONTINUITY

Figure 2 depicts the mass entering and leaving a control volume during time dt. Conservation of mass requires:

| Increase of mass within the control volume | = | Net mass flux through the control surface |
|---|---|---|

$$\frac{\partial}{\partial t}(\rho A dx)dt = \rho A u dt - [\rho A u + \frac{\partial}{\partial x}(\rho A u)dx]dt$$

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho A u) = 0$$

Since the duct is rigid, A is not a function of time but is a function only of x. Therefore expansion of the above equations results in:

$$\frac{\partial \rho}{\partial t} + \frac{\rho u dA}{A dx} + \rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} = 0$$

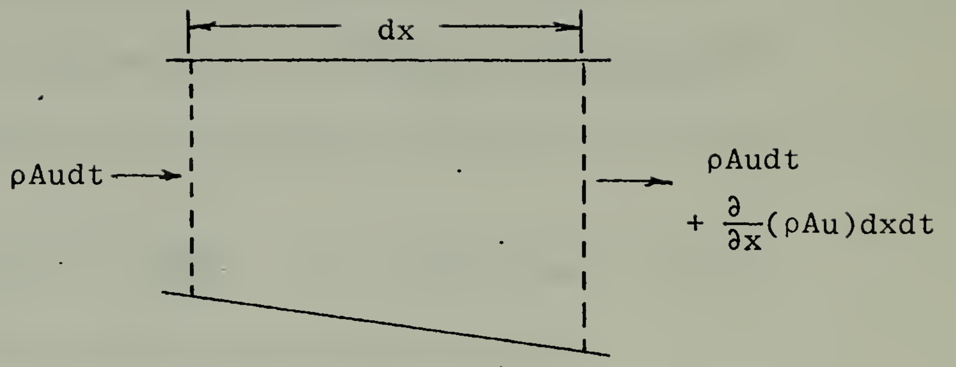


Figure 2. Mass Flow Through Control Volume During Time dt.

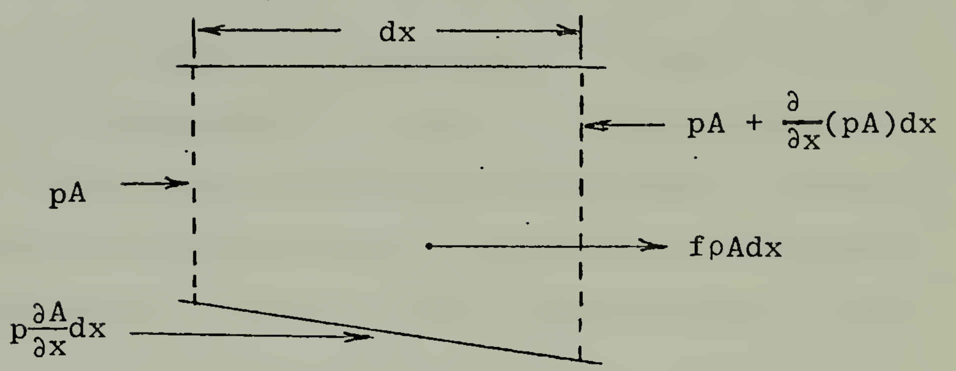


Figure 3. Forces Exerted on Control Volume

## B. MOMENTUM

Figure 3 depicts the forces on a control surface within a duct of varying area. From Newton's second law:

$$\text{body acceleration} = \frac{\text{forces acting on the body}}{\text{mass of the body}}$$

Let f = summation of frictional dissipative forces/unit mass; then

$$\frac{Du}{Dt} = \frac{1}{\rho A dx}\{pA + p\frac{\partial A}{\partial x}dx - [pA + \frac{\partial}{\partial x}(pA)dx] + f\rho A dx\}$$

Expanding the above equation, one gets

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{\rho A dx}[pA + p\frac{\partial A}{\partial x}dx - (pA + p\frac{\partial A}{\partial x}dx + A\frac{\partial p}{\partial x}dx) + f\rho A dx]$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f \quad .$$

## C. ENERGY

The preceding control volume approach may also be used for a similar derivation of the applicable energy equation. Several reliable references have used this approach and the results will not be duplicated here; exceptionally thorough discussions are contained in Ref. 3 and 8. For an adiabatic, quasi-one-dimensional flow of a perfect gas the energy equation reduces to:

$$\frac{\partial}{\partial t}(e + \frac{u^2}{2}) + u\frac{\partial}{\partial x}(e + \frac{u^2}{2}) + \frac{1}{\rho}\frac{\partial(pu)}{\partial x} + \frac{pu}{A\rho}\frac{dA}{dx} = 0$$

where e is the gas internal energy/unit mass.

## D. CHARACTERISTIC EQUATIONS

The equations of continuity, momentum and energy may be combined to obtain the characteristic (Riemann) wave equations.

Multiplication of the momentum equation by u yields

$$u\frac{\partial u}{\partial t} + u^2\frac{\partial u}{\partial x} + \frac{u}{\rho}\frac{\partial p}{\partial x} = fu \quad .$$

Subtracting the above from the energy equation, and assuming that f=0, gives

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial x} + \frac{p}{\rho}\frac{\partial u}{\partial x} + \frac{pu}{\rho A}\frac{dA}{dx} = 0 \ .$$

The continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\rho u}{A}\frac{dA}{dx} + \rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} = 0$$

may be re-written in the form

$$\frac{p}{\rho^2}\frac{\partial \rho}{\partial t} + \frac{pu}{\rho^2}\frac{\partial \rho}{\partial x} + \frac{p}{\rho}\frac{\partial u}{\partial x} + \frac{pu}{\rho A}\frac{dA}{dx} = 0 \ .$$

Subtracting this from the energy equation yields

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial x} - \frac{p}{\rho^2}\frac{\partial \rho}{\partial t} - \frac{pu}{\rho^2}\frac{\partial \rho}{\partial x} = 0 \ .$$

For an ideal gas with constant specific heats

$$d\left(\frac{s}{R}\right) = \frac{2}{\gamma-1}\frac{da}{a} - \frac{d\rho}{\rho} \quad .$$

For quasi-one-dimensional adiabatic flows the entropy of each particle may be expressed as

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} = 0 \quad .$$

Combination with the continuity equation produces

$$\frac{\partial a}{\partial t} + u\frac{\partial a}{\partial x} + a\frac{\partial}{\partial x}\left[\frac{\gamma-1}{2}\,u\right] = -\left(\frac{\gamma-1}{2}\right)ua\frac{dA}{dx} \ . \qquad (1)$$

The momentum equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

combined with the perfect gas relation

$$\frac{dp}{p} = \frac{2\gamma}{\gamma-1}\frac{da}{a} - \frac{ds}{R}$$

yields

$$\frac{\gamma-1}{2}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + a\frac{\partial a}{\partial x} - \frac{a^2(\gamma-1)}{2\gamma}\frac{\partial (s/R)}{\partial x} = 0$$

or

$$\frac{\gamma-1}{2}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + a\frac{\partial a}{\partial x} = \frac{a^2}{2c_p}\frac{\partial s}{\partial x} . \qquad (2)$$

Addition and subtraction of Equations 1 and 2 gives

$$\frac{\partial}{\partial t}\left(a + \frac{\gamma-1}{2}u\right) + (u+a)\frac{\partial}{\partial x}\left(a + \frac{\gamma-1}{2}u\right) = \frac{a^2}{2c_p}\frac{\partial s}{\partial x} - \frac{(\gamma-1)}{2}\frac{ua}{A}\frac{dA}{dx}$$

$$\frac{\partial}{\partial t}\left(a - \frac{\gamma-1}{2}u\right) + (u-a)\frac{\partial}{\partial x}\left(a - \frac{\gamma-1}{2}u\right) = -\frac{a^2}{2c_p}\frac{\partial s}{\partial x} - \frac{(\gamma-1)}{2}\frac{ua}{A}\frac{dA}{dx}$$

Note that the above two equations can be interpreted as the rate of change with time of the quantities $(a + \frac{\gamma-1}{2}u)$ and $(a - \frac{\gamma-1}{2}u)$ along lines with slopes of $\frac{1}{u+a}$ and $\frac{1}{u-a}$ respectively in the x, t plane. Defining new variables [Ref. 7]

$$U = \frac{\gamma-1}{2}u \qquad P = (p)^{\frac{\gamma-1}{2\gamma}}$$

$$\sigma = \exp[(s-s_*)/2c_p]$$

where $s_* = s_0 + 2c_p \ln a_0 - R \ln p_0$

which implies

$$P\sigma = a$$

the derived equations may be re-written

$$\frac{\partial}{\partial t}(P\sigma+U) + (u+a)\frac{\partial}{\partial x}(P\sigma+U) = \frac{a^2}{2c_p}\frac{\partial s}{\partial x} - \left(\frac{\gamma-1}{2}\right)\frac{ua}{A}\frac{dA}{dx}$$

$$\frac{\partial}{\partial t}(P\sigma-U) + (u-a)\frac{\partial}{\partial x}(P\sigma-U) = -\frac{a^2}{2c_p}\frac{\partial s}{\partial x} - \left(\frac{\gamma-1}{2}\right)\frac{ua}{A}\frac{dA}{dx}$$

Referring to Figure 4 and applying the above equations it can be seen

$$P_p\sigma_p+U_p = P_Q\sigma_Q+U_Q + \int_{pQ} \frac{a^2}{2c_p}\frac{\partial s}{\partial x} - \int_{pQ} (\frac{\gamma-1}{2})\frac{ua}{A}\frac{dA}{dx},$$

$$P_s\sigma_s-U_s = P_s\sigma_s-U_s - \int_{ps} \frac{a^2}{2c_p}\frac{\partial s}{\partial x} - \int_{ps} (\frac{\gamma-1}{2})\frac{ua}{A}\frac{dA}{dx}$$

which are the Riemann equations along the characteristics expressed in terms of the newly defined variables $P_s$, $\sigma_s$, and $U_s$. Reference 2 treats the constant area case of duct flow for which the above equations become

$$P_p\sigma_p+U_p = P_Q\sigma_Q+U_Q + \int_{pQ} \frac{a^2}{2c_p}\frac{\partial s}{\partial x}$$

$$P_s\sigma_s-U_s = P_s\sigma_s-U_s - \int_{ps} \frac{a^2}{2c_p}\frac{\partial s}{\partial x}$$

Through an analysis of wave interaction including a contact discontinuity, a justification is presented for writing the above equations as

$$P_p\sigma_Q+U_p = P_Q\sigma_Q+U_Q$$

$$P_p\sigma_s-U_p = P_s\sigma_s-U_s$$

and $\sigma_p$ = that value of $\sigma$ determined from the particle path of slope $1/u$.

Let HPS and HPQ represent the average values of $(\frac{\gamma-1}{2})\frac{ua}{A}\frac{dA}{dx}$ along the characteristics PS and PQ respectively. Then HPS dt and HPQ dt represent changes along the characteristics over time step dt due to the changing area. The term $(\frac{\gamma-1}{2})\frac{ua}{A}\frac{dA}{dx}$ may be easily obtained at points S and Q and an iterative procedure allows it to be evaluated at point P.

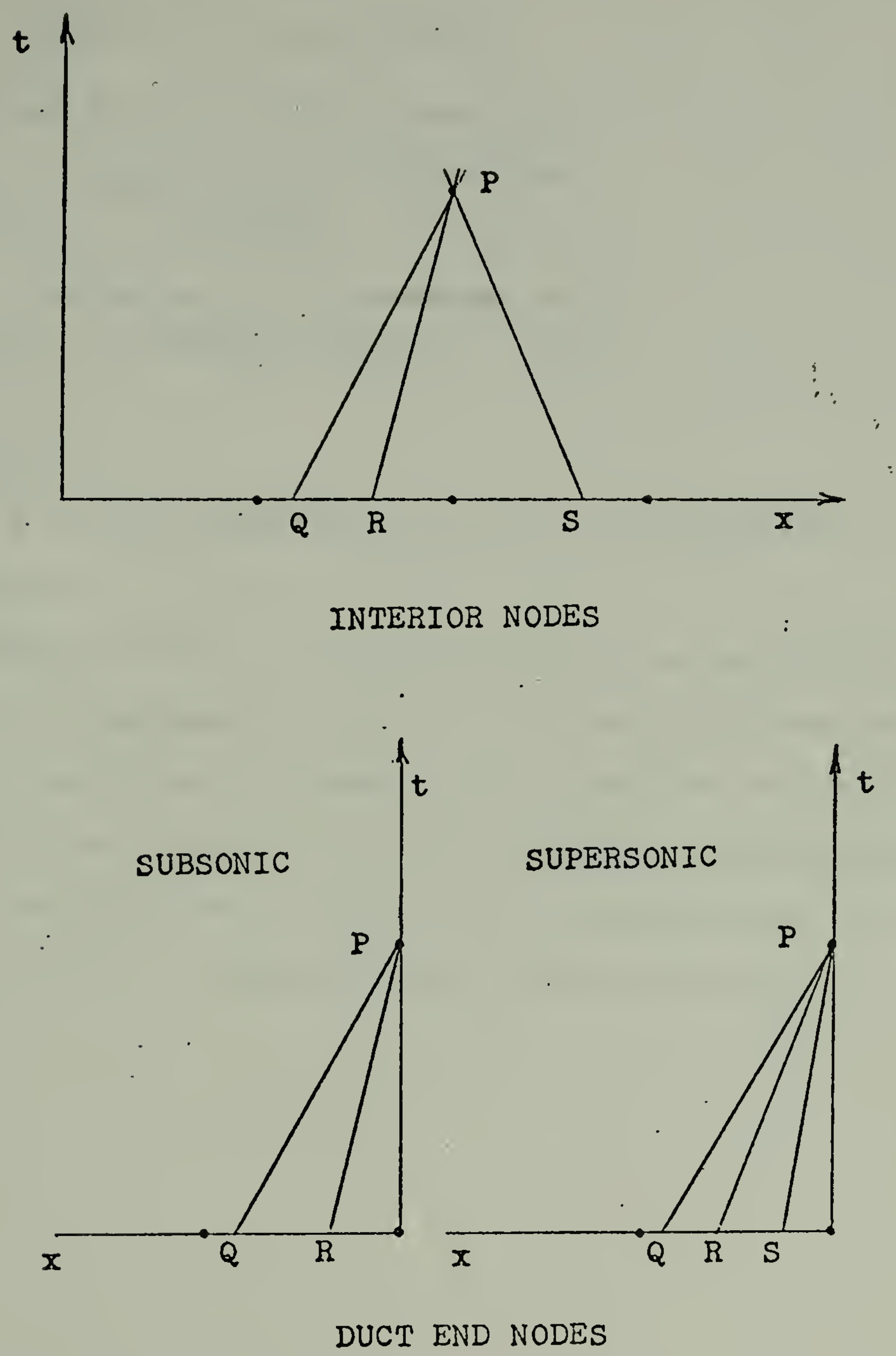


Figure 4. Computational Procedure.

The equations may now be re-written as

$$P_p\sigma_Q+U_p = P_Q\sigma_Q +U_Q-HPQ\ dt$$

$$P_p\sigma_s-U_p = P_s\sigma_s-U_s-HPS\ dt$$

which may be solved for $P_p$ and $U_p$ as

$$P_p = \frac{P_Q\sigma_Q+P_s\sigma_s+U_Q-U_s-HPQdt+HPSdt}{(\sigma_Q+\sigma_s)} \tag{3}$$

$$U_p = \frac{P_Q-P_s+U_Q/\sigma_Q+U_s/\sigma_s-HPQdt+HPSdt}{(1/\sigma_Q + 1/\sigma_s)} \tag{4}$$

$$\sigma_p = \sigma_R \tag{5}$$

where R is the x-intercept point of a characteristic having a slope 1/u.

Equations (3), (4), and (5) are the equations which are actually programmed. They are utilized in subroutine STEP and are modified by boundary conditions for use in determining duct end values in subroutine PORTS. A discussion of these modifications as well as how the equations are utilized is contained in a description of the program.

# IV. DESCRIPTION OF PROGRAM SS DIFFUSER

## A. PROGRAM EVOLUTION

The computer program developed is based on a program called PREX by Lebius Matthews [Ref. 7]. PREX (mnemonic for pressure exchanger) solves the unsteady flow problem resulting from opening and closing several valves which separate the duct from ports maintained at various pressure levels.

The valves utilized are modeled as orifices which change in size linearly with time during the opening and closing process. Extensive work was done in attempting to adapt this concept for application in program SS DIFFUSER (mnemonic for super-sonic diffuser). In this application extremely rapid valve operation was desired in order to simulate the high pressure rises obtained in laser starting. Additionally, the valves had to be re-modeled to handle subsonic and supersonic flow.

In all our attempts to use the valves in PREX, erroneous stagnation pressure increases were observed near the area of the valve. Once established, these new stagnation pressures then propagated into the nozzle/diffuser section. Additionally the magnitude of the established flow Mach number in the duct seemed to be a function of the valve opening time. The valves were finally abandoned in favor of a more simplified boundary condition which consists of a bursting diaphragm such as witnessed in shock tube applications. Subroutine PORTS was subsequently completely re-written to handle the

boundary conditions as outlined by Rudinger [Ref. 5]. Minor modifications were made in subroutine PORTX and in the main program, combined with the deletion of two other subroutines which handled the cyclic aspect of the pressure exchanger problem.

Most of the changes made to handle the variable area duct were made in subroutine STEP. This is also the subroutine which would have to be changed to include frictional or heating effects.

Subroutine STEP utilizes an entropy averaging process to avoid the discontinuity problems associated with shock waves. This is developed and discussed in depth in Ref. 6. The impact of this averaging on the results is that entropy values near the shock can not be relied upon to produce an accurate representation of the physical process. Also, if a stationary shock appears somewhere in the duct, the entropy gradient eventually averages out entirely.

## B. EXPLANATION OF SUBROUTINES

The following is an explanation of subroutines used in program SS DIFFUSER. Figure 5 contains a flow chart of the computational sequence. A list of symbols used in the computer program can be found in Appendix B.

### 1. BLOCK DATA (data input)

BLOCK DATA are divided into six categories of input data. Under FLUID PROPERTIES, the specific heat, $c_p$ and $c_v$, of the fluid are specified. These are treated as constants

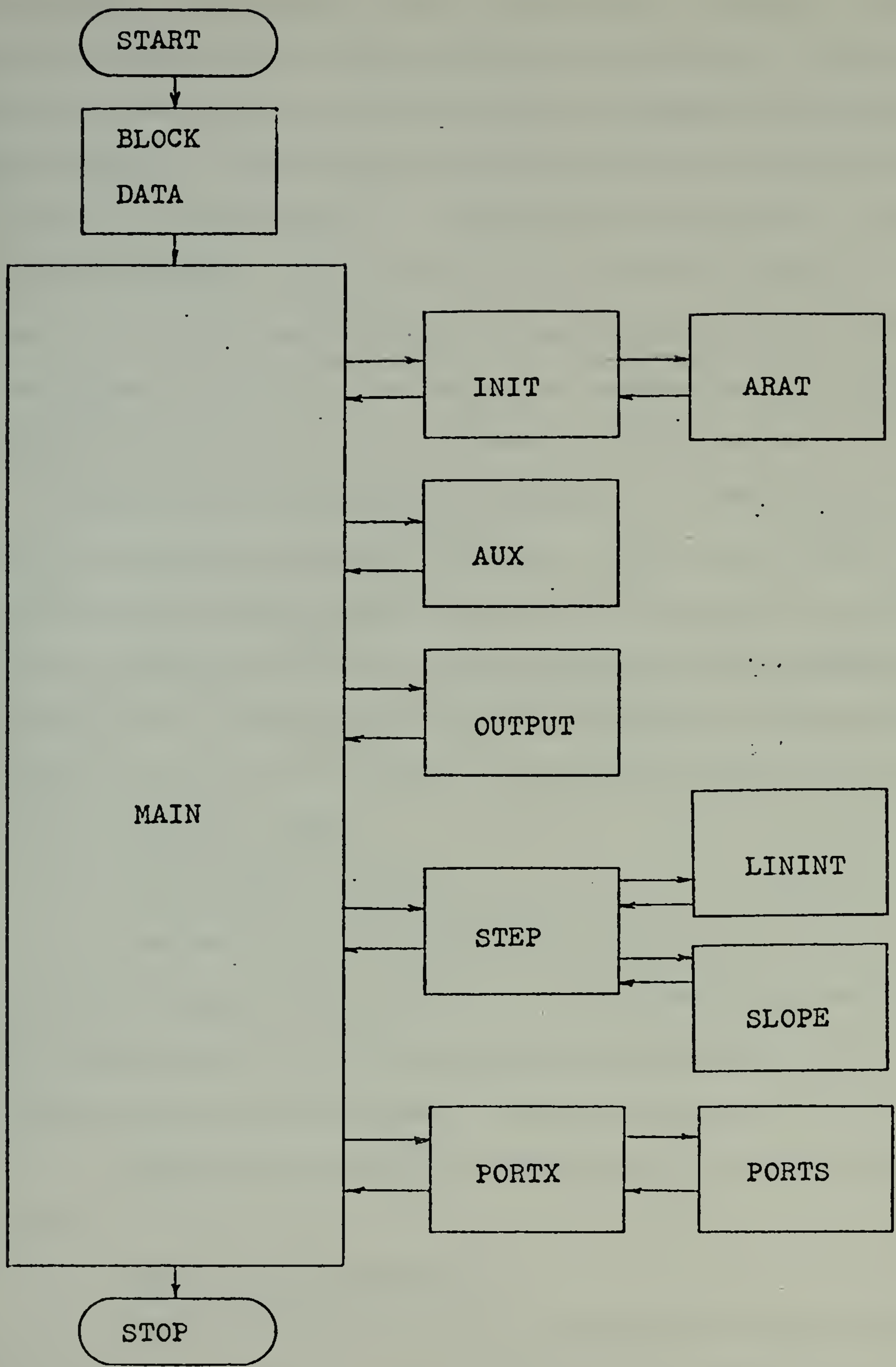


Figure 5. Flow Diagram of Program SS DIFFUSER.

throughout the program. Input data under BOUNDARY CONDITIONS are the temperatures and pressures at the ends of the duct. IDATA allows one to specify static or stagnation pressure input values. In the sample program these have been non-dimensionalized with respect to atmospheric conditions. In section VALVE PARAMETERS, the input value of NOVALV allows one to specify a fully open or fully closed right duct end. INITIAL CONDITIONS data fixes the values of temperature and pressure within the duct prior to the start of calculation.

DUCT PHYSICAL GEOMETRY information is used by subroutine ARAT to determine the area effect on the flow. A more detailed discussion of these values is included in the description of subroutine ARAT. PLOTTING AND PRINTING PARAMETERS controls the printing and plotting frequency and spacing of spatial nodes in the output. It also provides for a maximum time of solution, TMAX.

2. MAIN (main program)

The main program controls the order of calculations and specifically the calling of the designated subroutines. All the calculations are done on the subroutine level and passed to main through dummy variables and common statements.

MAIN utilizes TMAX to determine the maximum solution time.

3. INIT (initialization)

This subroutine utilizes the specific heat inputs to determine the GAM- constants (see Program Terminology, Appendix B) used throughout the program. It also establishes the

spatial interval according to the number of calculation points desired.

Slopes, areas, temperatures and pressures are initialized to the input values. From this information, initial values of the entropy variable, SIG, are determined.

4. ARAT (area ratio)

Subroutine ARAT utilizes the duct geometry to calculate the value of the area effect term in the Riemann Equations. Its output, RAT, is a ratio of the slope of the duct wall to the duct area at each value of x. Areas and x-coordinates have been non-dimensionalized with respect to duct length in the sample program.

To provide for a smooth change of area in the nozzle section, the area change has been modeled as a sinusoidal variation. The sinusoid amplitude, B, is equal to one-half the difference between the original duct area and the area at the nozzle throat. Thus, a smooth contour having zero slope at nozzle entrance, throat, and exit is provided.

The diffuser section allows for the modeling of a diffuser having an initial ramp input and three subsequent changes in flow direction. Figure 1 depicts the above models.

5. AUX (auxiliary)

Subroutine AUX calculates the values of output quantities other than at spatial calculation nodes. Although the values of A(I) and U(I) are used by subroutine STEP, other computations made here are not critical to calculations made by other routines. Their inclusion serves primarily as a check on input data and boundary conditions.

6. <u>OUTPUT (printed output)</u>

This routine gathers information from other subroutines and consolidates it for printing and plotting purposes. The plotting routine, UTPLOT, is a catalogued routine available under the IBM 360 system which was utilized. Similar such routines may be easily substituted.

The variables shown in the sample output have been non-dimensionalized with respect to atmospheric conditions. The entropy value (s) is actually s/R.

7. <u>STEP (time step)</u>

In a sense, STEP is the real controlling routine within the program. It controls the calculation at all interior points and uses routines SLOPE and LININT to do so. It essentially uses a backward linear interpolation scheme which may best be described by reference to Fig. 4.

Consider the calculation at some interior point P. At some previous time, $t_1$, the solution exists at points Q, R, and S as well as along the entire spatial grid. Since it is desired to maintain a constant grid interval, characteristics l, m, and n are generated through P based on the previously known values of U and A at points Q, R, and S. For the m characteristic shown, this would be the average of the values of U+A at Q and R. This same techniques is then applied along the entire length of duct.

The linear interpolation scheme requires that these characteristics intersect the x-axis at time $t_1$ somewhere between the calculation node and the adjacent node. To

facilitate this, the time step is adjusted to correspond with the minimum slope characteristic. Additionally, since this characteristic is only a first approximation, only a portion of this step is utilized as dictated by duct geometry and the expectation of area effect. It may be readily seen that the calculation time is quite sensitive to the number of grid points.

Once the time interval has been established, the corresponding x-intercepts are calculated by subroutine SLOPE. Values of the computing variables are then calculated at these intercepts by linear interpolation of the bracketing calculation nodes Q, R, and S. Using these values, a rough guess for the calculation variables are obtained for point P based on the Riemann equations with no area change.

The area effect term is integrated along the characteristic by assuming it to be a constant equal to the average of the end values. Subsequently new values are obtained at P through the introduction of this term. An iteration procedure is introduced here in order to obtain a more accurate area effect term.

Upon completion of the velocity and pressure calculation at point P, the entropy term, SIG, is updated at time t + dt.

8. SLOPE (characteristic slopes)

As previously mentioned, subroutine SLOPE is responsible for computing the slopes of the characteristics. Characteristics which pass through endpoints of the duct are

calculated only for the one originating within the duct. These are then combined with boundary values in subroutine PORTS (Fig. 4).

Two options may be used in calling SLOPE. The first option computes a slope on the basis of the average values of the bracketing spatial nodes. This is the option which is utilized in the sample program. The other option, which may be used with appropriate changes in subroutine STEP, allows the values in the slope equation to be externally dictated.

In either of the above options, the subsequent x-intercept at time t-dt is computed based on the slope and time interval.

9. <u>LININT (linear interpolation)</u>

LININT performs a simple linear interpolation for variables based on the values of the x-calculation nodes bracketing the desired spatial coordinate. The interpolation always takes place between two adjacent nodes. Endpoint exclusion is accomplished in the same manner as described for subroutine SLOPE.

10. <u>PORTX (port auxiliary)</u>

PORTX is the controlling routine for calculation of pressure and temperature variables external to the duct. If the duct ends are closed, the dummy variables passed to PORTS are appropriately set to zero. If the duct end is open, as in the case of the sample program, the variables are calculated based on whether static or stagnation values have been specified. These values are then passed to PORTS where they are meshed with the characteristic values.

11. PORTS (port solutions)

Subroutine PORTS controls the calculation of the velocity and pressure variables at the duct ends. If the ends are closed, PORTS provides the appropriate wave reflection from the Riemann equations.

For outflow, PORTS differentiates between two cases, sub-critical (subsonic) and super-critical (supersonic). For the sub-critical case the existing outside pressure is utilized as discussed in Ref. 5. For supersonic flow, all characteristics originate from within the duct and hence the Riemann equations are directly applicable. PORTS also provides an indicator to OUTPUT indicating whether the flow is supercritical or subcritical.

## V. DISCUSSION OF RESULTS

The computer program was initially used to solve a sample problem from Ref. 5. In Appendix A the computed solution is presented along with the graphical solution given by Ref. 5. The results can be seen to be in very good agreement with those of Rudinger.

Additional flows were considered utilizing nozzle contraction ratios of 4:1 and gas specific heat values $c_v$ and $c_p$ of 0.172 and 0.24 BTU/lbm-°R, respectively. Isentropic values for these conditions indicate a resulting Mach number of 2.94 should be obtained. Mach numbers of approximately 3.04 were obtained indicating an error of 3.4%. This resulting error is quite low considering the inherent limitations of the linear interpolation process utilized.

Tables I-V depict typical results obtained for a driver pressure value below that required to obtain a fully started condition. Time was nondimensionalized $(\frac{a_0 t}{L_0})$ utilizing a four foot duct length. The pressure utilized was $p/p_0$ of 16.0 ($p_0$ = ambient pressure). A semi-wedge diffuser (see Fig. 6) was utilized having a ramp slope of 0.05 ($\theta$=3°) and contraction ratio $(\frac{A_{\text{diffuser throat}}}{A_{\text{test section}}})$ of 0.94. Subsonic flow is established ahead of the nozzle throat (x=.8125) with supersonic flow in the diverging section. A formation of a stationary shock wave (Fig. 6) can be observed in the diverging section (x=.8700) of the nozzle resulting in subsonic flow in the test section (x=.8750 to x=.8900).

Tables VI-IX depict values for the same duct geometry for which the driver pressure has been elevated to 20.0 to drive the shock wave out of the nozzle and downstream (Fig. 7).

Pressures necessary for starting were considerably greater than witnessed in the laboratory. Possible explanations for this are that the program incorporates no friction, the bursting diaphragm causes a stagnation pressure loss, and only normal shocks are modeled. An effective stagnation pressure ratio of approximately 18.0 can be observed in the nozzle. When combined with the losses across a Mach 3.0 shock ($p_{t2}/p_{t1}$=.33), a correlation can be seen with laboratory observed starting values of 6.0.

These results appear to be in good agreement with the cited references. References 1 and 4 discuss similar conditions extensively. Although they treat the diverging section as a step increase in duct area, similar results are reported. For weak shock waves passing through a widening duct, a reflected expansion wave is created along with a transmitted shock. Additionally, a contact discontinuity is created by the entropy increase due to the shock. The resulting flow is subsonic in all regions; the entire process is shown in Figure 8A. For stronger shock waves region 5 disappears (Figure 8B), and at yet stronger values a stationary shock occurs as shown in Figure 8C. Further increases in arriving shock strength cause the stationary wave to be swept downstream as depicted in Figure 8D. Although these distinct waves can not be easily picked out in the computer results due to complications caused by a continuously varying area and the effects

of the nozzle contraction preceding in the area of interest, the general trends described above are in qualitative agreement. It appears as though the area contraction has a strengthening effect on the shock and causes an increase in shock velocity as might be expected from continuity. This eventually results in the formation of a reflected wave travelling upstream. Such an effect can be seen when a diffuser is added to a nozzle with the resulting larger pressure gradient across the stationary shock wave in the nozzle diverging section.

Figure 9 shows the effect of diffuser contraction ratio and ramp angle on start-up. For a fixed nozzle entrance to diffuser exit pressure ratio of 20.0, startups were attempted utilizing 56 combinations of ramp angle and contraction ratio. The results indicate that thin diffusers permit the use of higher ramp angles than do thick diffusers. Zerr [Ref. 8] determined similar results experimentally and advocates the use of thin, high ramp angle diffusers for efficient starting. Values for high ramp angles and extremely thin diffusers are not shown since these values slip through the computational grid size utilized (x=.01).

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTROPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 16.00000 | 16.00000 | 0.0 | 1.00000 | -2.77259 | 0.0 |
| 0.70000 | 16.00000 | 16.00000 | 0.0 | 1.00000 | -2.77259 | 0.0 |
| 0.71000 | 16.00000 | 16.00000 | 0.0 | 1.00000 | -2.77259 | 0.0 |
| 0.72000 | 16.00000 | 16.00000 | 0.0 | 1.00000 | -2.77259 | 0.0 |
| 0.73000 | 16.00000 | 16.00000 | 0.0 | 1.00000 | -2.77259 | 0.0 |
| 0.74000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.75000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.76000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.77000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.78000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.79000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.80000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.81000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.82000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.83000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.84000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.85000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.86000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.87000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.88000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.89000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.90000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.91000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.92000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.93000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.94000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.95000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.96000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.97000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.98000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.99000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 1.00000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |

Table I. Example 1: Initial Conditions

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTROPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 10.96072 | 12.20072 | 0.26396 | 0.89837 | -2.77259 | 0.27849 |
| 0.70000 | 9.37765 | 10.91025 | 0.36876 | 0.85953 | -2.77259 | 0.39775 |
| 0.71000 | 7.85226 | 9.60683 | 0.48524 | 0.81736 | -2.77259 | 0.53672 |
| 0.72000 | 6.39898 | 8.29621 | 0.61594 | 0.77131 | -2.77259 | 0.70133 |
| 0.73000 | 4.52420 | 6.47102 | 0.82883 | 0.69915 | -2.77259 | 0.99131 |
| 0.74000 | 3.68977 | 5.57469 | 0.94943 | 0.66188 | -2.76206 | 1.16701 |
| 0.75000 | 3.56274 | 5.34526 | 0.96998 | 0.66652 | -2.70234 | 1.18810 |
| 0.76000 | 4.70646 | 6.11982 | 0.82646 | 0.76905 | -2.47575 | 0.94242 |
| 0.77000 | 5.71062 | 6.05179 | 0.76057 | 0.94363 | -1.94713 | 0.78296 |
| 0.78000 | 6.58441 | 5.50964 | 0.75649 | 1.19507 | -1.25574 | 0.69200 |
| 0.79000 | 7.22059 | 4.98427 | 0.82490 | 1.44868 | -0.66876 | 0.68535 |
| 0.80000 | 7.16772 | 4.46574 | 1.00226 | 1.60505 | -0.29964 | 0.79111 |
| 0.81000 | 5.79402 | 3.62801 | 1.30710 | 1.59702 | -0.10456 | 1.03432 |
| 0.82000 | 4.27961 | 2.84705 | 1.31158 | 1.50317 | -0.01535 | 1.06977 |
| 0.83000 | 1.67020 | 1.44434 | 0.42251 | 1.15638 | -0.00015 | 0.39290 |
| 0.84000 | 1.03525 | 1.02514 | 0.02686 | 1.00986 | -0.00000 | 0.02673 |
| 0.85000 | 1.00107 | 1.00077 | 0.00081 | 1.00030 | 0.0 | 0.00081 |
| 0.86000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.87000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.88000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.89000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.90000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.91000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.92000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.93000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.94000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.95000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.96000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.97000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.98000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.99000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 1.00000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |

Table II. Example 1: Conditions at Time = .0599

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTROPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 7.04368 | 8.88704 | 0.55511 | 0.79258 | -2.77259 | 0.62353 |
| 0.70000 | 6.44887 | 8.34252 | 0.61105 | 0.77301 | -2.77259 | 0.69500 |
| 0.71000 | 5.87522 | 7.80371 | 0.66936 | 0.75288 | -2.77259 | 0.77144 |
| 0.72000 | 5.30513 | 7.25325 | 0.73238 | 0.73141 | -2.77259 | 0.85636 |
| 0.73000 | 4.47775 | 6.42333 | 0.83506 | 0.69711 | -2.77259 | 1.00016 |
| 0.74000 | 3.96537 | 5.88759 | 0.90715 | 0.67351 | -2.77259 | 1.10537 |
| 0.75000 | 3.75459 | 5.66154 | 0.93916 | 0.66317 | -2.77257 | 1.15325 |
| 0.76000 | 11.54157 | 12.65864 | 0.24492 | 0.91175 | -2.77202 | 0.25650 |
| 0.77000 | 12.38296 | 13.23900 | 0.23175 | 0.93534 | -2.75225 | 0.23963 |
| 0.78000 | 12.76644 | 13.18972 | 0.27262 | 0.96791 | -2.66194 | 0.27711 |
| 0.79000 | 12.79958 | 12.57313 | 0.37086 | 1.01801 | -2.48641 | 0.36757 |
| 0.80000 | 11.93055 | 11.28848 | 0.55282 | 1.05688 | -2.28386 | 0.53774 |
| 0.81000 | 7.73083 | 7.91009 | 0.95360 | 0.97734 | -2.12612 | 0.96459 |
| 0.82000 | 3.13795 | 4.01527 | 1.49918 | 0.78150 | -2.01370 | 1.69585 |
| 0.83000 | 1.52281 | 2.32869 | 1.76863 | 0.65393 | -1.91967 | 2.18711 |
| 0.84000 | 0.81557 | 1.45115 | 1.93611 | 0.56202 | -1.82987 | 2.58259 |
| 0.85000 | 0.51171 | 1.01269 | 2.04202 | 0.50530 | -1.73921 | 2.87268 |
| 0.86000 | 0.38245 | 0.80041 | 2.10538 | 0.47782 | -1.64542 | 3.04579 |
| 0.87000 | 1.68357 | 2.22257 | 1.24749 | 0.75749 | -1.50119 | 1.43334 |
| 0.88000 | 3.04906 | 3.10968 | 0.86282 | 0.98051 | -1.18432 | 0.87136 |
| 0.89000 | 3.08623 | 2.82307 | 0.85995 | 1.09322 | -0.81239 | 0.82247 |
| 0.90000 | 3.15130 | 2.60992 | 0.85257 | 1.20743 | -0.48253 | 0.77589 |
| 0.91000 | 3.16215 | 2.44850 | 0.86132 | 1.29147 | -0.24850 | 0.75792 |
| 0.92000 | 3.17611 | 2.36219 | 0.87136 | 1.34456 | -0.11071 | 0.75146 |
| 0.93000 | 3.18564 | 2.32118 | 0.88095 | 1.37242 | -0.04132 | 0.75198 |
| 0.94000 | 3.14095 | 2.27866 | 0.87611 | 1.37842 | -0.01180 | 0.74792 |
| 0.95000 | 2.84225 | 2.11533 | 0.80684 | 1.34364 | -0.00206 | 0.69606 |
| 0.96000 | 2.07731 | 1.68873 | 0.55203 | 1.23010 | -0.00014 | 0.49773 |
| 0.97000 | 1.34950 | 1.23963 | 0.21944 | 1.08864 | -0.00000 | 0.21032 |
| 0.98000 | 1.07603 | 1.05392 | 0.05284 | 1.02098 | -0.00000 | 0.05230 |
| 0.99000 | 1.01342 | 1.00960 | 0.01000 | 1.00378 | -0.00000 | 0.00998 |
| 1.00000 | 1.00000 | 1.00000 | 0.00313 | 1.00000 | -0.00000 | 0.00313 |

Table III. Example 1: Conditions at Time = .1438

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTROPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 6.48883 | 8.37953 | 0.60716 | 0.77437 | -2.77259 | 0.68996 |
| 0.70000 | 6.03244 | 7.95281 | 0.65291 | 0.75853 | -2.77259 | 0.74967 |
| 0.71000 | 5.58766 | 7.52804 | 0.70046 | 0.74225 | -2.77259 | 0.81304 |
| 0.72000 | 5.14004 | 7.09077 | 0.75171 | 0.72489 | -2.77259 | 0.88291 |
| 0.73000 | 4.49047 | 6.43640 | 0.83337 | 0.69767 | -2.77259 | 0.99773 |
| 0.74000 | 4.03393 | 5.96036 | 0.89706 | 0.67679 | -2.77259 | 1.09042 |
| 0.75000 | 3.82977 | 5.74258 | 0.92757 | 0.66691 | -2.77259 | 1.13583 |
| 0.76000 | 12.68567 | 13.54779 | 0.17713 | 0.93636 | -2.77255 | 0.18305 |
| 0.77000 | 12.85833 | 13.64408 | 0.19631 | 0.94241 | -2.76333 | 0.20222 |
| 0.78000 | 12.91162 | 13.51609 | 0.24697 | 0.95528 | -2.71961 | 0.25268 |
| 0.79000 | 12.74298 | 13.06676 | 0.34567 | 0.97522 | -2.63354 | 0.35004 |
| 0.80000 | 11.75635 | 11.98492 | 0.52173 | 0.98093 | -2.53236 | 0.52678 |
| 0.81000 | 7.73817 | 8.68105 | 0.88938 | 0.89139 | -2.45197 | 0.94201 |
| 0.82000 | 3.12664 | 4.45980 | 1.40712 | 0.70107 | -2.39342 | 1.68055 |
| 0.83000 | 1.53212 | 2.63717 | 1.65087 | 0.58097 | -2.34333 | 2.16589 |
| 0.84000 | 0.82778 | 1.67297 | 1.79829 | 0.49479 | -2.29433 | 2.55650 |
| 0.85000 | 0.52267 | 1.18624 | 1.88836 | 0.44061 | -2.24389 | 2.84484 |
| 0.86000 | 0.38845 | 0.94470 | 1.94437 | 0.41119 | -2.19102 | 3.03221 |
| 0.87000 | 0.33719 | 0.84010 | 1.97464 | 0.40137 | -2.13482 | 3.11687 |
| 0.88000 | 2.90580 | 3.83421 | 0.82413 | 0.75786 | -2.04526 | 0.94667 |
| 0.89000 | 2.97759 | 3.67280 | 0.81652 | 0.81071 | -1.83172 | 0.90684 |
| 0.90000 | 3.08813 | 3.47331 | 0.80401 | 0.88910 | -1.54241 | 0.85268 |
| 0.91000 | 3.10768 | 3.17258 | 0.81302 | 0.97954 | -1.20682 | 0.82147 |
| 0.92000 | 3.12457 | 2.89999 | 0.82320 | 1.07744 | -0.87604 | 0.79307 |
| 0.93000 | 3.13454 | 2.67899 | 0.83608 | 1.17005 | -0.58821 | 0.77294 |
| 0.94000 | 3.12812 | 2.51025 | 0.85427 | 1.24614 | -0.36379 | 0.76527 |
| 0.95000 | 3.09004 | 2.37969 | 0.87715 | 1.29851 | -0.20625 | 0.76975 |
| 0.96000 | 3.10519 | 2.32004 | 0.88290 | 1.33756 | -0.10592 | 0.76340 |
| 0.97000 | 3.08949 | 2.27531 | 0.89360 | 1.35783 | -0.04840 | 0.76687 |
| 0.98000 | 2.98763 | 2.20308 | 0.92637 | 1.35612 | -0.01935 | 0.79549 |
| 0.99000 | 2.66457 | 2.02231 | 1.02023 | 1.31759 | -0.00661 | 0.88881 |
| 1.00000 | 1.59097 | 1.39565 | 1.40798 | 1.13995 | -0.00205 | 1.31872 |

Table IV. Example 1: Conditions at Time = .1808

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTROPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 5.99458 | 7.91701 | 0.65684 | 0.75718 | -2.77259 | 0.75485 |
| 0.70000 | 5.65797 | 7.59581 | 0.69273 | 0.74488 | -2.77259 | 0.80264 |
| 0.71000 | 5.32618 | 7.27387 | 0.72995 | 0.73224 | -2.77259 | 0.85304 |
| 0.72000 | 4.98792 | 6.93973 | 0.77000 | 0.71875 | -2.77259 | 0.90825 |
| 0.73000 | 4.48486 | 6.43064 | 0.83411 | 0.69742 | -2.77259 | 0.99880 |
| 0.74000 | 4.10804 | 6.03864 | 0.88651 | 0.68029 | -2.77259 | 1.07458 |
| 0.75000 | 3.92114 | 5.84045 | 0.91375 | 0.67138 | -2.77259 | 1.11517 |
| 0.76000 | 12.88788 | 13.70242 | 0.16534 | 0.94056 | -2.77259 | 0.17049 |
| 0.77000 | 12.93281 | 13.72388 | 0.18970 | 0.94236 | -2.76931 | 0.19541 |
| 0.78000 | 12.91953 | 13.65344 | 0.24101 | 0.94625 | -2.75375 | 0.24776 |
| 0.79000 | 12.70407 | 13.37402 | 0.33733 | 0.94991 | -2.72331 | 0.34611 |
| 0.80000 | 11.68767 | 12.47009 | 0.50751 | 0.93726 | -2.68723 | 0.52422 |
| 0.81000 | 7.34357 | 8.86535 | 0.89268 | 0.82835 | -2.65850 | 0.98083 |
| 0.82000 | 3.15265 | 4.80694 | 1.35289 | 0.65585 | -2.63701 | 1.67055 |
| 0.83000 | 1.55237 | 2.87776 | 1.57960 | 0.53944 | -2.61825 | 2.15069 |
| 0.84000 | 0.84264 | 1.84767 | 1.71160 | 0.45605 | -2.59988 | 2.53451 |
| 0.85000 | 0.53618 | 1.32912 | 1.78827 | 0.40341 | -2.58072 | 2.81552 |
| 0.86000 | 0.40131 | 1.07364 | 1.83042 | 0.37375 | -2.56016 | 2.99392 |
| 0.87000 | 0.35094 | 0.96910 | 1.84945 | 0.36213 | -2.53788 | 3.07334 |
| 0.88000 | 2.80936 | 4.25987 | 0.79652 | 0.65949 | -2.50219 | 0.98083 |
| 0.89000 | 2.90604 | 4.27138 | 0.77972 | 0.68035 | -2.42613 | 0.94531 |
| 0.90000 | 3.06059 | 4.31000 | 0.75487 | 0.71012 | -2.32683 | 0.89580 |
| 0.91000 | 3.06835 | 4.16255 | 0.76158 | 0.73713 | -2.19757 | 0.88704 |
| 0.92000 | 3.06501 | 3.96957 | 0.77104 | 0.77213 | -2.03279 | 0.87748 |
| 0.93000 | 3.04173 | 3.72591 | 0.78570 | 0.81637 | -1.82849 | 0.86958 |
| 0.94000 | 2.98892 | 3.43852 | 0.80813 | 0.86925 | -1.58948 | 0.86678 |
| 0.95000 | 2.88484 | 3.11510 | 0.84055 | 0.92608 | -1.33050 | 0.87345 |
| 0.96000 | 2.83685 | 2.85743 | 0.85929 | 0.99280 | -1.06820 | 0.86240 |
| 0.97000 | 2.73303 | 2.59374 | 0.89380 | 1.05370 | -0.82078 | 0.87073 |
| 0.98000 | 2.56537 | 2.33258 | 0.94898 | 1.09980 | -0.60635 | 0.90489 |
| 0.99000 | 2.23146 | 2.01199 | 1.06281 | 1.10908 | -0.43724 | 1.00920 |
| 1.00000 | 1.82402 | 1.68134 | 1.22371 | 1.08486 | -0.31357 | 1.17487 |

Table V. Example 1: Conditions at Time = .2364

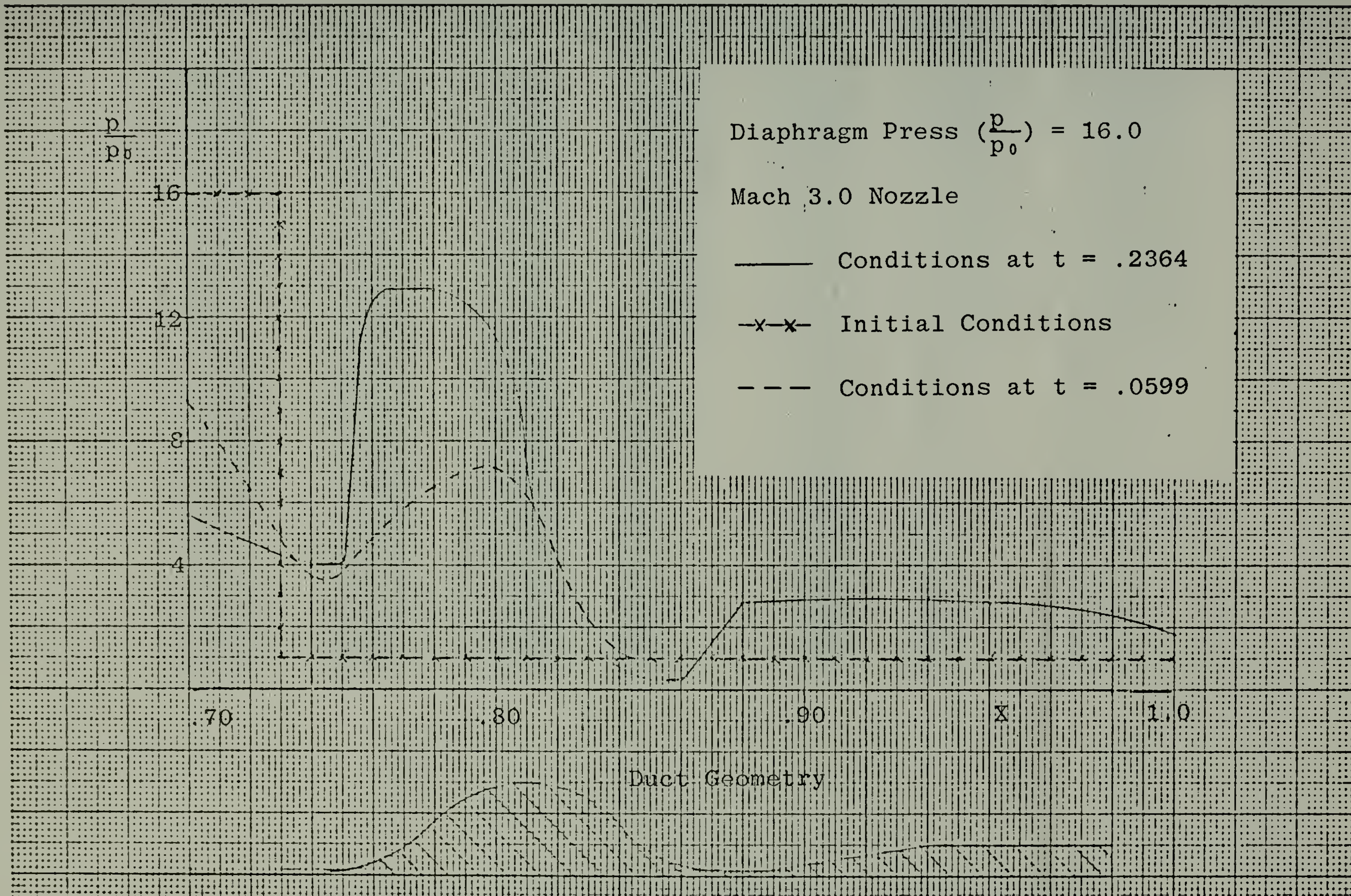


Figure 6. Example 1: Pressure Distribution at Time = .2364

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTRCPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 15.22607 | 16.44926 | 0.19172 | 0.92564 | -2.99573 | 0.19928 |
| 0.70000 | 12.95514 | 14.65127 | 0.30183 | 0.88423 | -2.99573 | 0.32098 |
| 0.71000 | 10.67559 | 12.75379 | 0.43048 | 0.83705 | -2.99573 | 0.47052 |
| 0.72000 | 8.48572 | 10.81896 | 0.57858 | 0.78434 | -2.99573 | 0.65330 |
| 0.73000 | 5.82598 | 8.26303 | 0.81102 | 0.70507 | -2.99573 | 0.96587 |
| 0.74000 | 4.20885 | 6.49427 | 1.00269 | 0.64809 | -2.96800 | 1.24551 |
| 0.75000 | 3.93240 | 5.97605 | 1.04238 | 0.65803 | -2.84634 | 1.28501 |
| 0.76000 | 4.73028 | 6.15685 | 0.95675 | 0.77080 | -2.47278 | 1.08975 |
| 0.77000 | 5.72260 | 5.74772 | 0.89619 | 0.99563 | -1.75988 | 0.89815 |
| 0.78000 | 6.42288 | 4.99533 | 0.91870 | 1.28578 | -0.97270 | 0.81020 |
| 0.79000 | 6.83152 | 4.44524 | 1.02568 | 1.53682 | -0.40491 | 0.82737 |
| 0.80000 | 6.60088 | 3.98044 | 1.25739 | 1.65833 | -0.10198 | 0.97641 |
| 0.81000 | 3.63556 | 2.52550 | 0.99068 | 1.43954 | -0.00492 | 0.82570 |
| 0.82000 | 1.19488 | 1.13610 | 0.13759 | 1.05174 | -0.00000 | 0.13416 |
| 0.83000 | 1.00612 | 1.00439 | 0.00477 | 1.00173 | 0.0 | 0.00477 |
| 0.84000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.85000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.86000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.87000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.88000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.89000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.90000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.91000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.92000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.93000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.94000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.95000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.96000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.97000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.98000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.99000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 1.00000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |

Table VI.  Example 2: Conditions at Time = .0505

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTROPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 12.31833 | 14.13147 | 0.33567 | 0.87169 | -2.99573 | 0.35953 |
| 0.70000 | 10.68427 | 12.76122 | 0.42995 | 0.83724 | -2.99573 | 0.46988 |
| 0.71000 | 9.12764 | 11.39940 | 0.53206 | 0.80071 | -2.99573 | 0.59460 |
| 0.72000 | 7.63238 | 10.02762 | 0.64535 | 0.76114 | -2.99573 | 0.73972 |
| 0.73000 | 5.65071 | 8.08411 | 0.82936 | 0.69899 | -2.99573 | 0.99200 |
| 0.74000 | 4.43667 | 6.79270 | 0.97188 | 0.65315 | -2.99324 | 1.20255 |
| 0.75000 | 4.06999 | 6.35978 | 1.02165 | 0.63996 | -2.97901 | 1.27710 |
| 0.76000 | 5.10131 | 7.34336 | 0.91745 | 0.69468 | -2.91526 | 1.10075 |
| 0.77000 | 7.73044 | 9.26350 | 0.72038 | 0.83451 | -2.68369 | 0.78858 |
| 0.78000 | 9.54593 | 9.34175 | 0.67274 | 1.02186 | -2.17979 | 0.66550 |
| 0.79000 | 10.83941 | 8.55545 | 0.71086 | 1.26696 | -1.54806 | 0.63154 |
| 0.80000 | 10.99511 | 7.40047 | 0.85590 | 1.48573 | -1.00013 | 0.70219 |
| 0.81000 | 7.70434 | 5.16776 | 1.26430 | 1.49085 | -0.63233 | 1.03546 |
| 0.82000 | 4.11832 | 3.08099 | 1.72698 | 1.33669 | -0.39123 | 1.49373 |
| 0.83000 | 3.21199 | 2.44448 | 1.71595 | 1.31398 | -0.20316 | 1.49696 |
| 0.84000 | 3.68848 | 2.59167 | 1.32833 | 1.42321 | -0.05964 | 1.11345 |
| 0.85000 | 2.52186 | 1.94267 | 0.76986 | 1.29814 | -0.00406 | 0.67570 |
| 0.86000 | 1.21907 | 1.15254 | 0.14870 | 1.05773 | -0.00001 | 0.14458 |
| 0.87000 | 1.01342 | 1.00960 | 0.00965 | 1.00378 | -0.00000 | 0.00963 |
| 0.88000 | 1.00052 | 1.00037 | 0.00037 | 1.00015 | 0.0 | 0.00037 |
| 0.89000 | 1.00001 | 1.00001 | 0.00001 | 1.00000 | 0.0 | 0.00001 |
| 0.90000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.91000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.92000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.93000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.94000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.95000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.96000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.97000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.98000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 0.99000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |
| 1.00000 | 1.00000 | 1.00000 | 0.0 | 1.00000 | 0.0 | 0.0 |

Table VII. Example 2: Conditions at Time = .0724

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTROPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 7.96066 | 10.33487 | 0.61894 | 0.77027 | -2.99573 | 0.70523 |
| 0.70000 | 7.42785 | 9.83430 | 0.66230 | 0.75530 | -2.99573 | 0.76207 |
| 0.71000 | 6.90682 | 9.33486 | 0.70737 | 0.73990 | -2.99573 | 0.82236 |
| 0.72000 | 6.38040 | 8.81928 | 0.75596 | 0.72346 | -2.99573 | 0.88878 |
| 0.73000 | 5.60419 | 8.03635 | 0.83431 | 0.69735 | -2.99573 | 0.99909 |
| 0.74000 | 4.99005 | 7.39492 | 0.90321 | 0.67479 | -2.99573 | 1.09952 |
| 0.75000 | 4.67399 | 7.05615 | 0.94155 | 0.66240 | -2.99573 | 1.15687 |
| 0.76000 | 16.07944 | 17.10482 | 0.16707 | 0.94005 | -2.99573 | 0.17231 |
| 0.77000 | 16.24418 | 17.22251 | 0.18728 | 0.94319 | -2.99414 | 0.19284 |
| 0.78000 | 16.25736 | 17.13300 | 0.23839 | 0.94889 | -2.97370 | 0.24473 |
| 0.79000 | 16.00965 | 16.70717 | 0.33615 | 0.95825 | -2.92371 | 0.34339 |
| 0.80000 | 14.75153 | 15.47075 | 0.50972 | 0.95351 | -2.85936 | 0.52199 |
| 0.81000 | 8.63496 | 10.38104 | 0.94867 | 0.83180 | -2.80580 | 1.04018 |
| 0.82000 | 3.94131 | 5.85091 | 1.37462 | 0.67362 | -2.76593 | 1.67484 |
| 0.83000 | 1.94153 | 3.48802 | 1.60947 | 0.55663 | -2.73122 | 2.15725 |
| 0.84000 | 1.05444 | 2.23000 | 1.75014 | 0.47284 | -2.69653 | 2.54517 |
| 0.85000 | 0.66902 | 1.59302 | 1.83463 | 0.41997 | -2.66006 | 2.83099 |
| 0.86000 | 0.49953 | 1.27787 | 1.88597 | 0.39091 | -2.62101 | 3.01646 |
| 0.87000 | 0.43412 | 1.14181 | 1.91394 | 0.38021 | -2.57866 | 3.10397 |
| 0.88000 | 0.41598 | 1.09303 | 1.93307 | 0.38058 | -2.53252 | 3.13347 |
| 0.89000 | 0.41171 | 1.06958 | 1.94867 | 0.38492 | -2.48212 | 3.14088 |
| 0.90000 | 0.65235 | 1.46013 | 1.75150 | 0.44678 | -2.41646 | 2.62038 |
| 0.91000 | 3.39546 | 4.53706 | 0.86790 | 0.74838 | -2.24540 | 1.00325 |
| 0.92000 | 3.41949 | 4.11473 | 0.87779 | 0.83104 | -1.88272 | 0.96289 |
| 0.93000 | 3.43306 | 3.69318 | 0.89101 | 0.92957 | -1.49123 | 0.92415 |
| 0.94000 | 3.41179 | 3.30438 | 0.91077 | 1.03251 | -1.11434 | 0.89632 |
| 0.95000 | 3.32523 | 2.95512 | 0.94102 | 1.12524 | -0.78507 | 0.88711 |
| 0.96000 | 3.33870 | 2.74805 | 0.94830 | 1.21493 | -0.51845 | 0.86034 |
| 0.97000 | 3.31611 | 2.58467 | 0.96445 | 1.28377 | -0.31776 | 0.85121 |
| 0.98000 | 3.20185 | 2.42326 | 1.00448 | 1.32130 | -0.18038 | 0.87386 |
| 0.99000 | 2.70321 | 2.09650 | 1.15290 | 1.28939 | -0.09737 | 1.01531 |
| 1.00000 | 1.93097 | 1.62656 | 1.42973 | 1.18715 | -0.05254 | 1.31221 |

Table VIII. Example 2: Conditions at Time = .1926

| X | PRESSURE | DENSITY | VELOCITY | TEMP | ENTROPY | MACH |
|---|---|---|---|---|---|---|
| 0.69000 | 7.60951 | 10.00608 | 0.64723 | 0.76049 | -2.99573 | 0.74218 |
| 0.70000 | 7.16145 | 9.58023 | 0.68499 | 0.74752 | -2.99573 | 0.79227 |
| 0.71000 | 6.72053 | 9.15372 | 0.72419 | 0.73419 | -2.99573 | 0.84518 |
| 0.72000 | 6.27186 | 8.71149 | 0.76641 | 0.71995 | -2.99573 | 0.90325 |
| 0.73000 | 5.59478 | 8.02668 | 0.83532 | 0.69702 | -2.99573 | 1.00053 |
| 0.74000 | 5.05764 | 7.46657 | 0.89528 | 0.67737 | -2.99573 | 1.08779 |
| 0.75000 | 4.76743 | 7.15696 | 0.92959 | 0.66612 | -2.99573 | 1.13947 |
| 0.76000 | 16.17359 | 17.18037 | 0.16239 | 0.94169 | -2.99573 | 0.16734 |
| 0.77000 | 16.22142 | 17.20860 | 0.18702 | 0.94263 | -2.99484 | 0.19263 |
| 0.78000 | 16.18758 | 17.13117 | 0.23865 | 0.94492 | -2.98421 | 0.24551 |
| 0.79000 | 15.90307 | 16.79111 | 0.33520 | 0.94711 | -2.95829 | 0.34443 |
| 0.80000 | 14.62183 | 15.66093 | 0.50546 | 0.93365 | -2.92482 | 0.52311 |
| 0.81000 | 9.11967 | 11.07881 | 0.89464 | 0.82316 | -2.89726 | 0.98606 |
| 0.82000 | 3.93949 | 6.03458 | 1.34905 | 0.65282 | -2.87619 | 1.66968 |
| 0.83000 | 1.93960 | 3.61244 | 1.57523 | 0.53692 | -2.85743 | 2.14975 |
| 0.84000 | 1.05409 | 2.32116 | 1.70716 | 0.45412 | -2.83877 | 2.53331 |
| 0.85000 | 0.67179 | 1.67134 | 1.78402 | 0.40194 | -2.81904 | 2.81395 |
| 0.86000 | 0.50373 | 1.35153 | 1.82668 | 0.37271 | -2.79767 | 2.99211 |
| 0.87000 | 0.44109 | 1.22074 | 1.84635 | 0.36133 | -2.77430 | 3.07157 |
| 0.88000 | 0.42550 | 1.18104 | 1.85624 | 0.36028 | -2.74864 | 3.09255 |
| 0.89000 | 0.42008 | 1.16095 | 1.86793 | 0.36184 | -2.72053 | 3.10530 |
| 0.90000 | 0.41931 | 1.14937 | 1.87730 | 0.36482 | -2.68976 | 3.10810 |
| 0.91000 | 0.44272 | 1.18340 | 1.86655 | 0.37411 | -2.65532 | 3.05169 |
| 0.92000 | 0.86828 | 1.88942 | 1.57041 | 0.45955 | -2.60290 | 2.31657 |
| 0.93000 | 2.96569 | 4.34973 | 0.92611 | 0.68181 | -2.43869 | 1.12158 |
| 0.94000 | 3.32966 | 4.31608 | 0.87129 | 0.77145 | -2.11867 | 0.99199 |
| 0.95000 | 3.18372 | 3.79463 | 0.91182 | 0.83901 | -1.77759 | 0.99547 |
| 0.96000 | 3.18188 | 3.44393 | 0.92223 | 0.92391 | -1.43680 | 0.95945 |
| 0.97000 | 3.12545 | 3.10079 | 0.94472 | 1.00795 | -1.11163 | 0.94098 |
| 0.98000 | 2.99509 | 2.77241 | 0.98644 | 1.08032 | -0.82430 | 0.94906 |
| 0.99000 | 2.62821 | 2.36477 | 1.09540 | 1.11140 | -0.59352 | 1.03905 |
| 1.00000 | 2.09740 | 1.91788 | 1.27690 | 1.09360 | -0.42489 | 1.22103 |

Table IX. Example 2: Conditions at Time = .2243

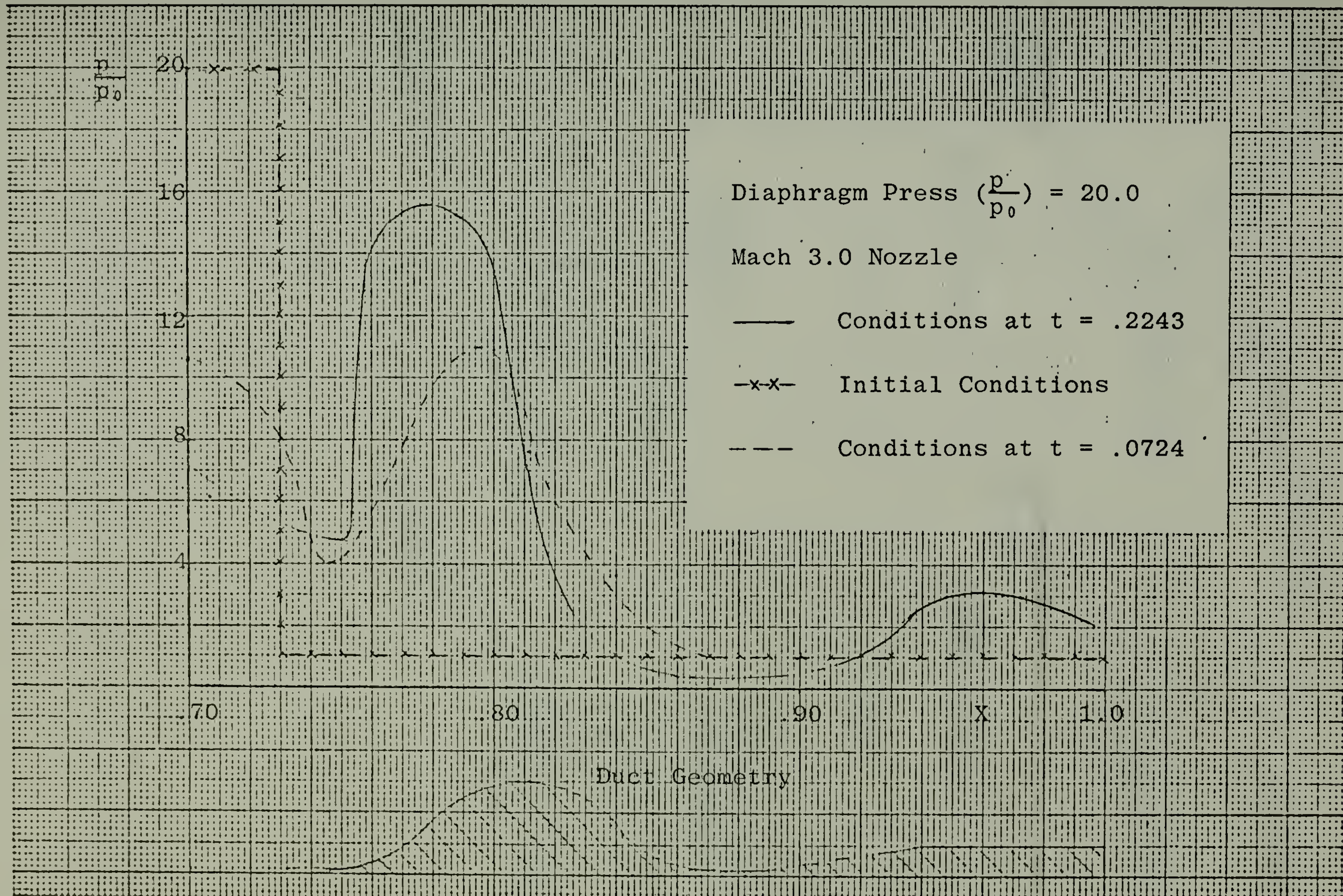


Figure 7. Example 2: Pressure Distribution at Time = .2243

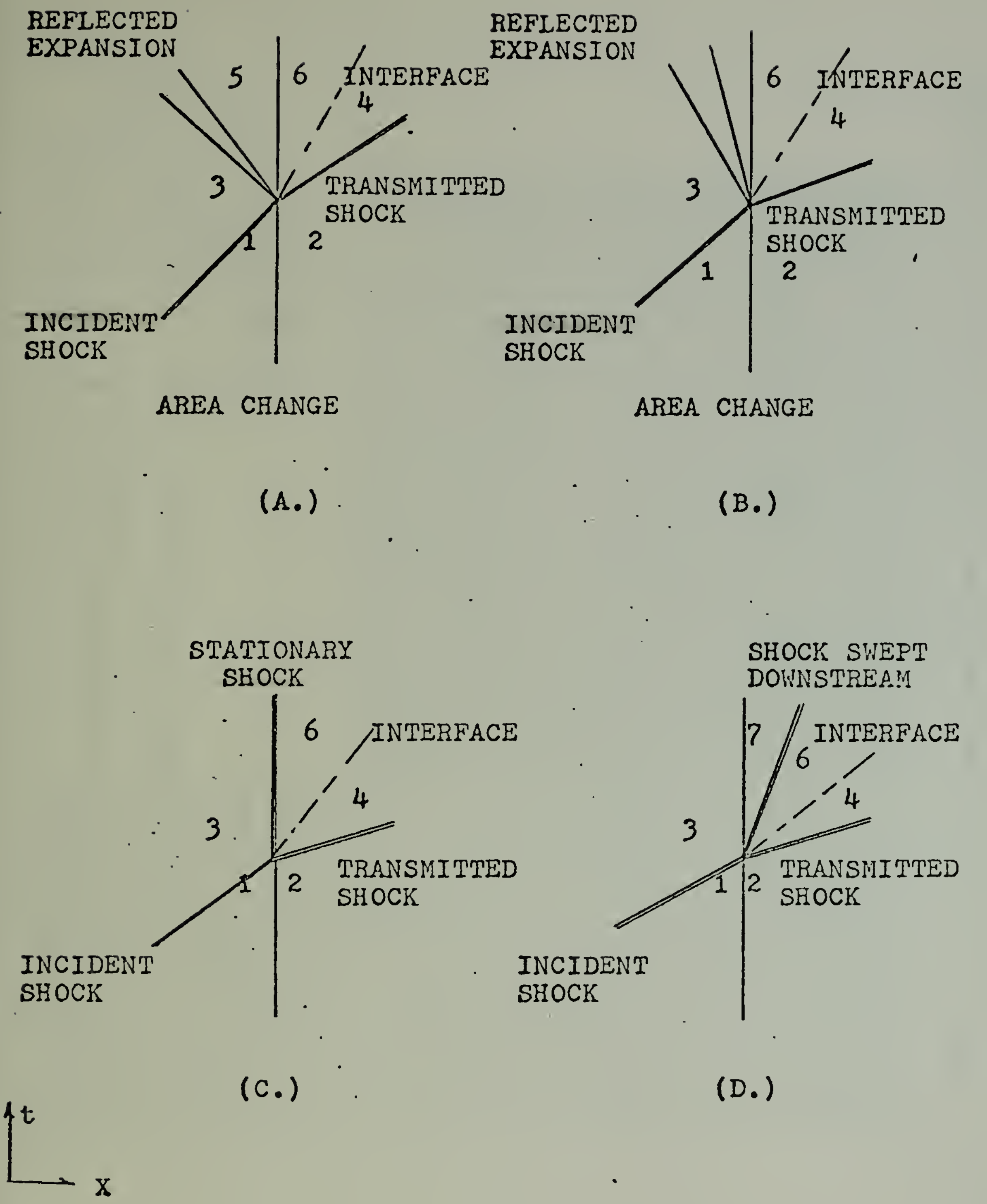


Figure 8. Waves Resulting From Shock Wave Passing Through a Widening of the Duct as a Function of Increasing Pressure Ratio.

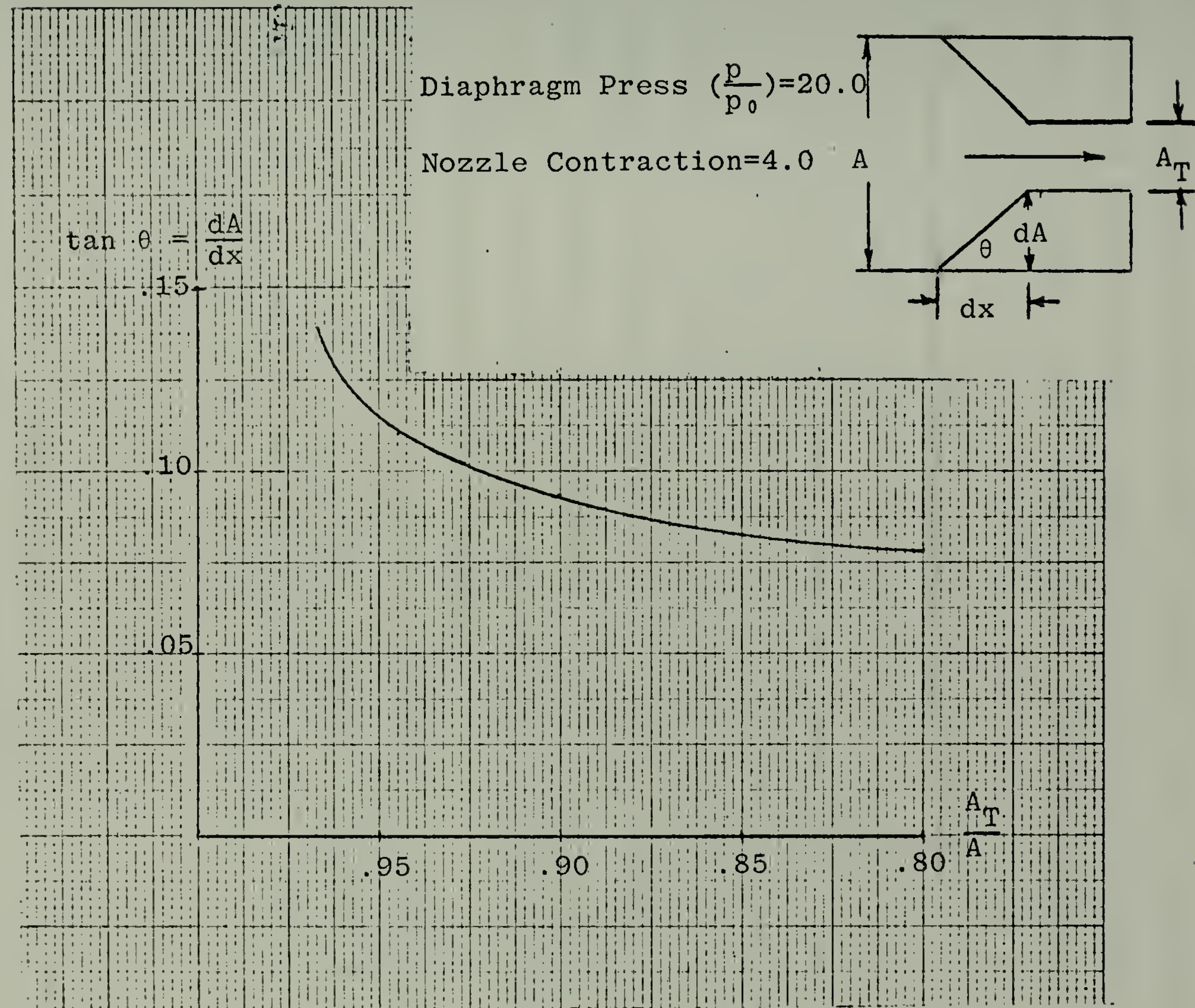


Figure 9. Effects of Diffuser Ramp Angle and Contraction Ratio on Starting.

## VI. CONCLUSIONS AND RECOMMENDATIONS

The developed computer model appears to give satisfactory results in one-dimensional wave analysis of the diffuser starting phenomena. Only a few of the possible parameters have been varied thus far but the program may be used for more extensive studies in a number of areas.

The high contraction ratio nozzle did not appear to violate the implicit quasi-one-dimensional assumption. Reference 1 gives the criterion that the axial component of the flow velocity and its derivatives should be larger than the corresponding transverse component by at least one order of magnitude, to satisfy this assumption. While this criterion may be overly restrictive in some applications, it must be abided by in high contraction ratio problems.

The program appears to have no difficulties in the transonic range where one of the characteristics develops a vertical slope. A problem was anticipated in this regime, and slope and interpolation equations were written so as to avoid indeterminate forms.

One of the most restrictive features of the computer program is the time step dependence on grid size. The interpolation process requires that the characteristics at time step t + dt originate within one spatial grid space at time t. Thus in decreasing the grid size for more accuracy one must pay a high price in calculating time.

As previously mentioned, extensions may be made in subroutine STEP to include frictional and heat transfer effects. Additional changes may be possible to include other boundary layer phenomena.

Studies so far have been based on diffusers with a leading ramp angle followed by a constant area section to the duct end. A similar study may be done on a full wedge diffuser (converging/diverging) in order to gain a more complete picture of diffuser effects.

Reference 2 documents the results of several applications of the basic techniques utilized by the program. These include a shock tube, supercharger and pressure exchanger. Program SS DIFFUSER may be used for constant area shock tube problems by setting the diffuser wedge angles and nozzle sinusoid equal to zero and utilizing the closed end condition for the right end.

## APPENDIX A

## RUDINGER DUCT PROBLEM

The following problem was taken from Ref. 4. The computed output is presented for comparison with the graphical solution presented by Ref. 4.

PROBLEM:

A duct of constant cross section is filled with air ($\gamma=1.4$) that is isentropically compressed from atmospheric pressure to $P_1=2.83$. The duct is suddenly opened to the atmosphere. How does the pressure vary at the closed end of the duct?

SOLUTION:

The pressure history at the duct end is depicted in Figure A1. Close agreement is seen throughout. Initially an expansion wave travels toward the closed end and is reflected back to the open end. The open end condition causes it to reflect back as a shock wave which reaches the closed end shortly after $\frac{a_0 t}{L_0} = 3.0$.

There is a slight variation of results in the vicinity of the shock wave where the computer solution depicts a finite slope instead of the predicted vertical discontinuity. This is a result of the averaging process across the shock previously mentioned.

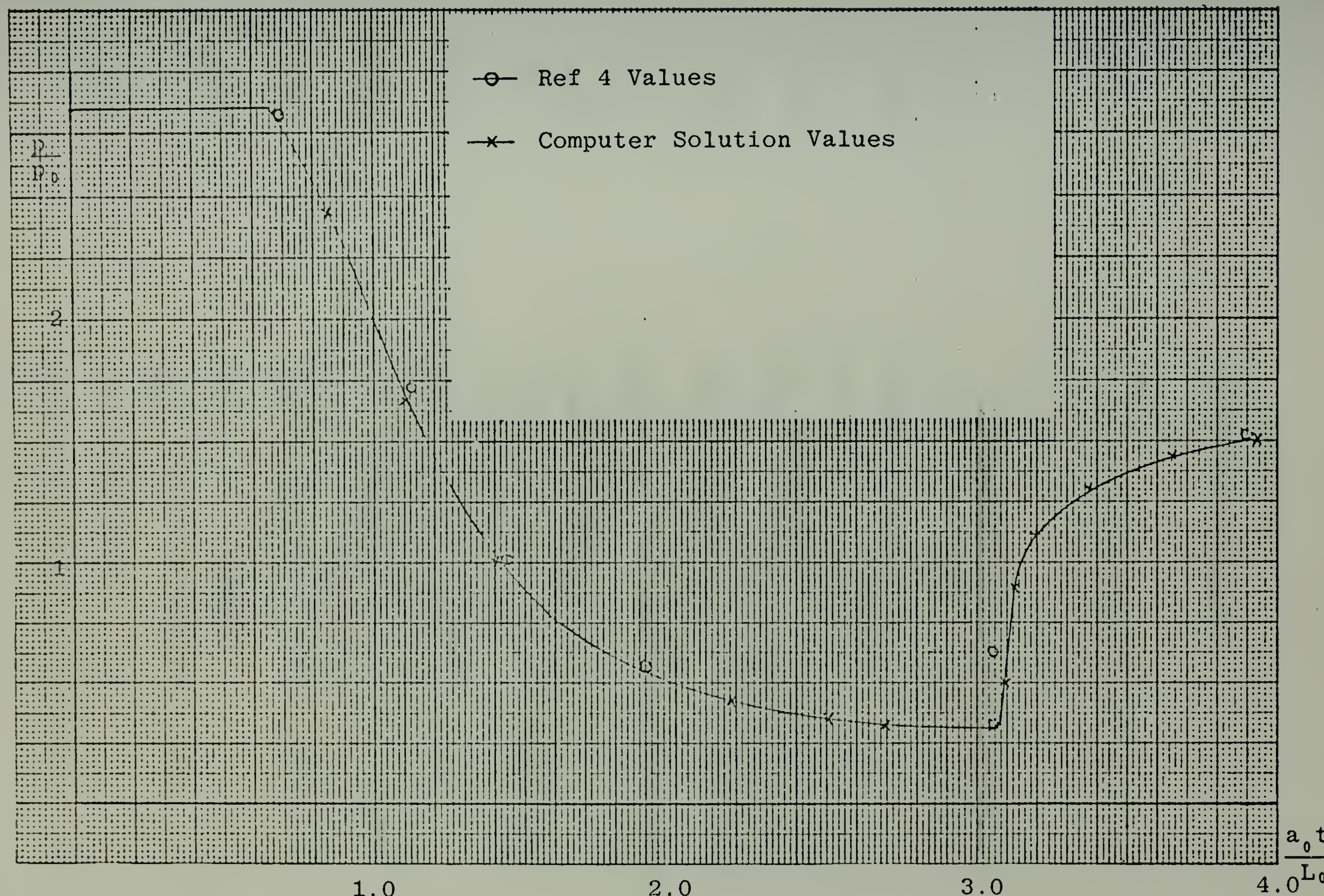


Figure A1. Rudinger Duct Problem Results

## APPENDIX B

## PROGRAM TERMINOLOGY

| Fortran Symbol | | Meaning |
|---|---|---|
| A(I) | a | Sonic velocity |
| AA | | Duct area |
| AK | | Fraction of maximum allowable time step |
| AM(I) | U | Velocity variable |
| AMAC(I) | | Local Mach number=U/A |
| AM1 | | Initial 'AM' in duct |
| AM1(I) | | 'AM' interpolated along 'N' characteristic |
| AM2(I) | | 'AM' interpolated along 'M' characteristic |
| AMT | | 'AM' prior to inclusion of area effect term |
| AN(I) | | Pressure variable |
| AN1 | | Initial value of 'AN' in duct (low pressure side) |
| AN1I | | Initial value of 'AN' in duct (high pressure side) |
| AN1(I) | | 'AN' interpolated along 'N' characteristic |
| AN2(I) | | 'AN' interpolated along 'M' characteristic |
| AREA | | Duct area (in ARAT) |
| ANT | | 'AN' prior to inclusion of area effect term |
| AUX | | Subroutine name |
| B | | Amplitude of nozzle sinusoidal area variation |

| | | |
|---|---|---|
| BE(I) | | Characteristic slope |
| BEM(I) | $B_m$ | Slope of 'M' characteristic |
| BEN(I) | $B_n$ | Slope of 'N' characteristic |
| BES(I) | $B_s$ | Slope of 'S' characteristic |
| BMAX | | Maximum of BEM, BEN, and BES |
| C | | Label for common block |
| C1 | | Label for common block |
| C2 | | Label for common block |
| C3 | | Label for common block |
| CELL | | Label for common block |
| COMP | | Label for common block |
| CP | $c_p$ | Specific heat at constant pressure |
| CV | $c_v$ | Specific heat at constant volume |
| CYC | | Label for common block |
| DADX | | Da/dx change in duct area with respect to x |
| DG | | Increment in 'G' |
| DL | | Duct length |
| DT | | Time step size |
| DX | | Spatial step size |
| EPS | | Iteration control term |
| FLOW | | Label for common block |
| FRA | | Fraction in linear interpolation |
| G(NPORT) | | Integrated mass flow into port numbered 'NPORT' |
| GAM | | Ratio of specific heats=CP/CV |
| GAMC | | Critical pressure ratio for INFLOW=SQRT((GAS+1)/2) |

| | | |
|---|---|---|
| GAMMA | | Label for common block |
| GAM1 | | = (GAM-1)/2 |
| GAM2 | | = -2/GAM1 |
| GAM3 | | = 2(GAM-1)/(GAM+1) |
| GAM4 | | = (2/(GAM+1))**((GAM+1)/(2*(GAM-1))) |
| GAM5 | | = GAM/GAM1 |
| GAM6 | | = 1/GAM5 |
| GAM7 | | = (GAM+1)/(2+GAM) |
| GAM8 | | = (GAM+1)/(GAM-1) |
| HPS | | Area effect term along 'M' characteristic |
| HPQ | | Area effect term along 'N' characteristic |
| IDATA(I) | $\text{INDEX}_1$ | = 1 If port static pressure supplied |
| | | = 2 If port stagnation pressure supplied |
| IHAND | | Index identifying port 1 for left hand port, and 2 for right |
| IND | | Indicator to route computation 1 for NC flow-----closed port 8 for super-critical outflow through fully open port 9 for sub-critical outflow through fully open port |
| INDL | | 'IND' at left end |
| INDR | | 'IND' at right end |
| INIT | | Subroutine name |
| IOPEN | | Index, =2 if end closed, =1 if end open |
| IP | | Alphameric symbol identifying end |
| IR | | Alphameric symbol for right end |
| J | | Counter |
| K | | Counter in DO loop |
| K1 | | KDPM + 1 |

| | | |
|---|---|---|
| KDPM | | Location of diaphragm in duct |
| L | | Causes print of variables every L-TH node |
| LININT | . | Subroutine name |
| MAIN | | The main program |
| N | | Numbers of grid intervals in spatial direction |
| N1 | | Numbers of grid points in spatial direction = N+1 |
| NOPENL | | Number of port open on left hand side |
| NOPENR | | Number of port open on right hand side |
| NOVALV | | NOVALV = 1 Implies right end open, closed for all other values |
| NPLOT | | Plot control |
| NPRINT | | Print control |
| NSTEP | | Number of time steps |
| OUTPUT | | Subroutine name |
| P(I) | $p$ | Pressure |
| PHI | | Duct end opening (if open, if closed) |
| PHI | | Sinusoidal angle in ARAT |
| PHIL | | 'PHI' at left port |
| PHIR | | 'PHI' at right port |
| PI | | Initial pressure in duct (low pressure side) |
| PII | | Initial pressure in duct (high pressure side) |
| PIE | | 3.1415926536----- |
| PO | $p_{st}$ | Stagnation pressure |
| POCL | | Stagnation pressure in duct, left boundary |

| | | |
|---|---|---|
| POCR | | Stagnation pressure at duct, right boundary |
| POL | | Stagnation pressure at left end (external) |
| POP(I) | | Stagnation pressure at duct end (external) |
| POR | | Stagnation pressure at right end (external) |
| PORTS | | Subroutine name |
| PORTX | | Subroutine name |
| POS | | Non-dimensional spatial coordinate |
| PR | | Ratio of stagnation to static pressure |
| PRL | | 'PR' at left end (external) |
| PRR | | 'PR' at right end (external) |
| PROP | | Label for common block |
| PX | $p_{ex}$ | Static pressure (external) |
| PXL | | Static pressure at left end (external) |
| PXP(I) | | Static pressure in port 'I' |
| PXR | | Static pressure at right end (external) |
| PXS | | Dummy argument for 'PXL' or 'PXR' |
| RAN(I) | | Limits of output plot |
| RANGE | | Label for common block |
| RATIO | | Label for common block |
| RAT(I) | | Dummy variable in ARAT for $\frac{1da}{A\ dx}$ |
| RATX(I) | | RAT evaluated at C(I) |
| RATX1(I) | | RAT interpolated along 'N' characteristic |
| RATX2(I) | | RAT interpolated along 'M' characteristic |
| RO(I) | $\rho$ | Specific density |
| S(I) | $s$ | Entropy |

| | | |
|---|---|---|
| S(I) | | Dummy variable in linear interpolation |
| S1(I) | | Dummy variable in linear interpolation |
| SIG(I) | | Dependent variable, function of entropy |
| SIG1(I) | | SIG interpolated along 'N' characteristic |
| SIG2(I) | | SIG interpolated along 'M' characteristic |
| SIGI | | Initial value of SIG (low pressure side) |
| SIGII | | Initial value of SIG (high pressure side) |
| SL | | Slope in linear interpolation |
| SLOPE | | Subroutine name |
| SOL | | Stagnation entropy at left end (external) |
| SOR | | Stagnation entropy at right port |
| START | | Label for common block |
| STEP | | Subroutine name |
| T(I) | T | Temperature |
| THETA(I) | | Angles in diffuser geometry |
| TI | | Initial temperature in duct (low pressure side) |
| TII | | Initial temperature in duct (high pressure side) |
| TIM | t | Normalized time |
| TMAX | | Cycle time |
| TO | | Stagnation temperature |
| TOCL | | Stagnation temperature in duct, left boundary |
| TOCR | | Stagnation temperature in duct, right boundary |
| TOL | | Stagnation temperature at left end (external) |
| TOP(I) | | Stagnation temperature at duct end |

| | | |
|---|---|---|
| TOR | | Stagnation temperature at right end (external) |
| U(I) | u | Velocity |
| X | | Dummy variable |
| X(I) | x | Spatial distance |
| X1(I) | | X interpolated along BEN |
| X2(I) | | X interpolated along BEM |
| X3(I) | | X interpolated along BES |

```
C          **************************************************
C          *                                                *
C          *              PROGRAM   SSDIFFUSER              *
C          *                                                *
C          **************************************************
C        COMPUTER PROGRAM FOR CALCULATING TIME DEPENDENT FLOW           C
C             THROUGH A SUPERSONIC NOZZLE/DIFFUSER                      C
C            BY  K.D.  AANERUD                       JUNE 1974          C
      BLOCK DATA
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 RANGE,RAN
      COMMON/PROP/CP,CV
      COMMON/STATUS/NOVALV
      COMMON/CELL/XI,PII,PI,TI,TII
      COMMON/COMP/AK,N,L,NPRINT,NPLOT
      COMMON/RATIO/RATX(101),RATX1(101),RATX2(101),RAT(101)
      COMMON/GEOM/POS(6),THETA(4),AA,B,CL,KDPM
      COMMON/RANGE/RAN(4)
      COMMON/CYC/TOP(2),PXP(2),POP(2),IDATA(2),TMAX
C
C     FLUID PROPERTIES
      DATA CP,CV/0.24,0.172/
C
C     BOUNDARY CONDITIONS
      DATA IDATA/2,1/
      DATA TOP/1.0,1.0/
      DATA PXP/2.,1.0/
      DATA POP/1.0,1.0/
C
C     VALVE PARAMETERS
      DATA NOVALV/1/
C
C     INITIAL CONDITIONS
      DATA XI,PI,PII,TI,TII/1.0,1.0,20.0,1.0,1.0/
C
C     DUCT PHYSICAL GEOMETRY
      DATA AA,B,DL,KDPM/4.D0,1.5D0,48.D0,74/
      DATA THETA/.050D0,0.D0,0.D0,0.D0/
      DATA POS/.75D0,.875D0,.89D0,.9400D0,1.D0,0.D0/
C
C     PLOTTING AND PRINTING PARAMETERS
      DATA TMAX/.300/
      DATA AK,N,L,NPRINT/1.0,100,1,02/
      DATA RAN/1.0,0.,20.0,0./
      DATA NPLOT/100/
      END
```

```
C       MAIN PROGRAM
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON /FLOW/ G(2),DG(2),TOCL,TOCR,POCL,POCR
        COMMON /COMP/ AK,N,L,NPRINT,NPLOT
        COMMON /GEOM/ POS(6),THETA(4),AA,B,DL,KDPM
        COMMON /CYC/ TOP(2),PXP(2),POP(2),IDATA(2),TMAX
C
C       INITIALIZATION
        CALL INIT (DX,N)
        N1=N+1
        TIM=0.0
        INDL=1
        INDR=1
C
        DO 1 NPORT=1,2
        DG(NPORT)=1.E-14
      1 G(NPORT)=1.E-14
C
        NOPENL=0
        NSTEP=0
C
C       COMMENCE TIME STEPS
      2 CALL AUX (N1,DT,NOPENL,NOPENR)
C
C       PRINT RESULTS OF DESIGNATED STEPS
        IF(MOD(NSTEP,NPRINT).EQ.0) CALL OUTPUT(N1,TIM,INDL,PHIL,PXL,POL,
       1TCL,NSTEP,INDR,PHIR,PXR,POR,TOR)
        NSTEP=NSTEP+1
        CALL STEP (AK,DX,DT,N1)
        TIM=TIM+DT
        CALL PORTX (N1,INDL,NOPENL,PHIL,PXL,POL,TOL,INDR,NOPENR,PHIR,PXR,P
       1OR,TOR)
C
C       COMPARE CURRENT TIME WITH MAXIMUM DESIRED TIME
        IF (TIM.LT.TMAX) GO TO 2
C
        CALL OUTPUT (N1,TIM,INDL,PHIL,PXL,POL,TOL,NSTEP,INDR,PHIR,PXR,POR,
       1TOR)
        STOP
        END
```

```
      SUBROUTINE INIT (DX,N)
C     CALCULATES CONSTANTS,DETERMINES NODES AND SETS INITIAL CONDS
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /U/ AM(101),AN(101),SIG(101),X(101)
      COMMON /GAMMA/ GAM,GAM1,GAM2,GAM3,GAMC,GAM4,GAM5,GAM6
      COMMON /PROP/ CP,CV
      COMMON /CYC/ TOP(2),PXP(2),POP(2),IDATA(2),TMAX
      COMMON /C1/ AM1(101),AN1(101),SIG1(101)
      COMMON /C2/ AM2(101),AN2(101),SIG2(101)
      COMMON /CELL/ XI,PII,PI,TI,TII
      COMMON /RATIO/ RATX(101),RATX1(101),RATX2(101),RAT(101)
      COMMON /GEOM/ POS(6),THETA(4),AA,B,DL,KDPM
C
C     EVALUATE THE GAM- CONSTANTS TO BE USED IN CALCULATIONS
      GAM=CP/CV
      GAM1=(GAM-1.)/2.
      GAM2=-2./GAM1
      GAM3=2.*(GAM-1.)/(GAM+1.)
      GAMC=DSQRT((GAM+1.)/2.)
      GAM4=(2./(GAM+1.))**((GAM+1.)/2./(GAM-1.))
      GAM5=GAM/GAM1
      GAM6=1./GAM5
      GAM7=(GAM+1.D0)/(GAM*2.D0)
      GAM8=(GAM-1.D0)/(GAM+1.D0)
C
C     ESTABLISH SPATIAL CALCULATION NODES
      N1=N+1
      DX=1./DFLOAT(N)
      X(1)=0.0
C
      DO 1 I=1,N
    1 X(I+1)=X(I)+DX
C
C
C     DETERMINE SLOPE/AREA RATIOS AT CALCULATION NODES
      CALL ARAT (X,N,RATX)
C
C     INITIAL CELL DISTRIBUTION
      K1=KDPM+1
      ANI=PI**GAM6
      ANII=PII**GAM6
      SIGI=DSQRT(TI)/ANI
      SIGII=DSQRT(TII)/ANII
C
      DO 2 I=1,KDPM
      AN(I)=ANII
      AM(I)=0.D0
```

```
      SIG(I)=SIGII                                                      IN 490
      SIG2(I)=SIGII                                                     IN 500
      SIG1(I)=SIGII                                                     IN 510
    2 CONTINUE                                                          IN 520
C                                                                       IN 530
C                                                                       IN 540
      DO 3 I=K1,N1                                                      IN 550
      AN(I)=ANI                                                         IN 560
      AM(I)=0.                                                          IN 570
      SIG(I)=SIGI                                                       IN 580
      SIG2(I)=SIGI                                                      IN 590
      SIG1(I)=SIGI                                                      IN 600
    3 CONTINUE                                                          IN 610
C                                                                       IN 620
      RETURN                                                            IN 630
      END                                                               IN 640
```

```
      SUBROUTINE ARAT (X,N,RAT)
C     CALCULATES RATIO OF SLOPE/AREA AT POSITION X
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /GEOM/ POS(6),THETA(4),AA,B,DL,KDFM
      DIMENSION RAT(101), X(101)
C
      N1=N+1
      PIE=3.141592653b
      AR=AA/DL
      T=(AA-2.DO*B)/DL
C
      DO 7 I=1,N1
      IF (X(I).LT.POS(1)) GO TO 1
      IF (X(I).LT.POS(2)) GO TO 2
      IF (X(I).LT.POS(3)) GO TO 1
      IF (X(I).LT.POS(4)) GO TO 3
      IF (X(I).LT.POS(5)) GO TO 4
      IF (X(I).LT.POS(6)) GO TO 5
    1 AREA=AR
      DADX=0.DO
      GO TO 6
    2 PHI=2.DO*PIE*(X(I)-POS(1))/(POS(2)-POS(1))
      DADX=-B*2.DO*PIE/DL/(POS(2)-POS(1))*DSIN(PHI)
      AREA=T+B/DL+B/DL*DCOS(PHI)
      GO TO 6
    3 DADX=-2.DO*THETA(1)
      AREA=AR+DADX*(X(I)-POS(3))
      GO TO 6
    4 DADX=-2.DO*THETA(2)
      AREA=AR-2*THETA(1)*(POS(4)-POS(3))+DADX*(X(I)-POS(4))
      GO TO 6
    5 DADX=2.DO*THETA(3)
      AREA=AR+DADX*(X(I)-POS(6))
    6 RAT(I)=DADX/AREA
    7 CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE OUTPUT (N1,TIM,INDL,PHIL,PXL,PCL,TOL,NSTEP,INDR,PHIR,PX
     1R,PCR,TCR)
C     CONTROLS PRINTING OF RESULTS
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL *4RANGE,RAN,RENGE,REN
      COMMON /C/ AM(101),AN(101),SIG(101),X(101)
      COMMON /C3/ P(101),A(101),AMAC(101),RO(101),T(101),S(101),U(101)
      COMMON /GAMMA/ GAM,GAM1,GAM2,GAM3,GAM4,GAM5,GAM6
      COMMON /FLOW/ G(2),DG(2),TOCL,TOCR,POCL,POCR
      COMMON /COMP/ AK,N,L,NPRINT,NPLOT
      COMMON /RANGE/ RAN(4)
      DATA IL/'LEFT'/
      DATA IR/'RITE'/
C
      IF (NSTEP.EQ.0) WRITE (6,19)
      IHAND=0
    1 IHAND=IHAND+1
      GO TO (2,3,14), IHAND
C                   LEFT HAND PORT
    2 IP=IL
      GO TO (4,5,6,7,8,9,10,11,12,13,1), INDL
C                   RIGHT HAND PORT
    3 IP=IR
      GO TO (4,5,6,7,8,9,10,11,12,13,1), INDR
    4 WRITE (6,23) IP
      GO TO 1
    5 WRITE (6,24) IP
      GO TO 1
    6 WRITE (6,25) IP
      GO TO 1
    7 WRITE (6,26) IP
      GO TO 1
    8 WRITE (6,27) IP
      GO TO 1
    9 WRITE (6,28) IP
      GO TO 1
   10 WRITE (6,29) IP
      GO TO 1
   11 WRITE (6,30) IP
      GO TO 1
   12 WRITE (6,31) IP
      GO TO 1
   13 WRITE (6,32) IP
      GO TO 1
   14 CONTINUE
      IF (NSTEP.EQ.0) GO TO 15
C
```

```
      SOL=0.
      SOR=0.
      IF (INDL.NE.1) SOL=DLOG(TOL**(GAM5/2.)/POL)
      IF (INDR.NE.1) SOR=DLOG(TOR**(GAM5/2.)/POR)
C
      WRITE (6,20) G(1)
      WRITE (6,35)
      WRITE (6,21) TOCL,TOCR,POCL,POCR
      WRITE (6,33) NSTEP,PHIL,PXL,POL,TCL,SOL,TIM,PHIR,PXR,POR,TOR,SOR
C
   15 WRITE (6,34)
      WRITE (6,22) (X(I),P(I),RO(I),U(I),T(I),S(I),AMAC(I),I=1,N1,L)
      WRITE (6,18)
      IF (MOD(NSTEP,NPLOT).NE.0) GO TO 16
      WRITE (6,17) TIM
      CALL UTPLOT (X,P,N1,RAN,2,1)
      CALL UTPLOT (X,T,N1,RAN,2,3)
      CALL UTPLOT (X,AMAC,N1,RAN,2,1)
      CALL UTPLOT (X,U,N1,RAN,2,3)
   16 RETURN
C
C
   17 FORMAT (1X,' PRESSURE,MACH, AND TEMP PROFILES AT TIME',F15.7,/)
   18 FORMAT (75H0**********************************************************
     1****************,39H***************************************,/)
   19 FORMAT (1H1)
   20 FORMAT (1X,'MASS FLOW IN=',F15.4)
   21 FORMAT (12X,11HTEMPERATURE,19X,2F10.4,/,12X,8HPRESSURE,22X,2F10.4,
     1/)
   22 FORMAT(7F11.5)
   23 FORMAT (43H0                   NO  FLOW       -       CLOSED,A6,5H PORT)
   24 FORMAT (43H0SUB   -CRITICAL  INFLOW THROUGH PARTLY OPEN,A6,5H PORT)
   25 FORMAT (43H0SUPER-CRITICAL  INFLOW THROUGH PARTLY OPEN,A6,5H PORT)
   26 FORMAT (43H0SUB   -CRITICAL  INFLOW THROUGH  FULLY OPEN,A6,5H PORT)
   27 FORMAT (43H0SUPER-CRITICAL  INFLOW THROUGH  FULLY OPEN,A6,5H PORT)
   28 FORMAT (43H0SUB   -CRITICAL OUTFLOW THROUGH PARTLY OPEN,A6,5H PORT)
   29 FORMAT (43H0SUPER-CRITICAL OUTFLOW THROUGH PARTLY OPEN,A6,5H PORT)
   30 FORMAT (43H0SUPER-CRITICAL OUTFLOW THROUGH  FULLY OPEN,A6,5H PORT)
   31 FORMAT (43H0SUB   -CRITICAL OUTFLOW THROUGH  FULLY OPEN,A6,5H PORT)
   32 FORMAT (43H0SUB   -CRITICAL OUTFLOW THROUGH PARTLY OPEN,A6,5H PORT)
   33 FORMAT (5H0STEP,I5,5X,4HPHIL,F7.4,2X,3HPXL,F7.4,2X,4HPOL ,F7.4,2X,
     14HTOL ,F7.4,2X,4HSOL ,F7.4,/,5H TIME,F8.4,2X,5HPHIR ,F6.4,2X,4HPXR
     2 ,F6.4,2X,5HPOR  ,F6.4,2X,5HTOR  ,F6.4,2X,5HSOR  ,F6.4,/)
   34 FORMAT(/,8X,1HX,07X,1HP,10X,2HRO,09X,1HU,10X,1HT,10X,1HS,08X,  4HM
     1CH,/)
   35 FORMAT(41H0STAGNATION QUANTITIES AT CELL BOUNDARIES,6X,
     C    4HLEFT,5X,5HRIGHT )
      END
```

```
      SUBROUTINE LININT (BE,DX,X,X3,N1,S,S1)
C     BACKWARD LINEAR INTERPOLATION
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S(101), S1(101), X(101), X3(101), BE(101)
C
C     DETERMINE IF ENDPOINTS ARE TO BE INCLUDED
      L=1
      N=N1-1
      IF (BE(1).GT.0.0) L=2
      IF (BE(N1).GT.0.0) N=N1
C
C     INTERPOLATE BETWEEN GRID POINTS
C
      DO 1 I=L,N
      K=I+1
      IF (BE(I).GT.0.0) K=I-1
      FRA=DABS(X3(I)-X(I))/DX
    1 S1(I)=(1.-FRA)*S(I)+FRA*S(K)
C
      RETURN
      END
```

```
      SUBROUTINE AUX (N1,DT,NOPENL,NOPENR)
C     CALCULATES AUXILIARY VARIABLES
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /C/ AM(101),AN(101),SIG(101),X(101)
      COMMON /C3/ P(101),A(101),AMAC(101),RO(101),T(101),S(101),U(101)
      COMMON /FLOW/ G(2),DG(2),TOCL,TOCR,POCL,POCR
      COMMON /GAMMA/ GAM,GAM1,GAM2,GAM3,GAMC,GAM4,GAM5,GAM6
C
C     CALCULATE OUTPUT QUANTITIES
C
      DO 1 I=1,N1
      P(I)=AN(I)**GAM5
      A(I)=AN(I)*SIG(I)
      U(I)=AM(I)/GAM1
      AMAC(I)=U(I)/A(I)
      T(I)=A(I)**2
      S(I)=DLOG(SIG(I))*GAM5
    1 RO(I)=P(I)/T(I)
C
C
C     CONVENTION,POSITIVE VELOCITY INDICATES FLOW TO RIGHT
C
C     CELL BOUNDARY STAGNATION QUANTITIES
      TOCL=(1.+(GAM-1.)/2.*AMAC(1)**2)*T(1)
      TOCR=(1.+(GAM-1.)/2.*AMAC(N1)**2)*T(N1)
      POCL=TOCL**(GAM/(GAM-1.))/DEXP(S(1))
      POCR=TOCR**(GAM/(GAM-1))/DEXP(S(N1))
C
C     INTEGRATING FLOW BY TRAPEZOIDAL RULE
C     CONVENTION. POSITIVE MASS FLOW INDICATES FLOW INTO CELL
      IF (NOPENL.EQ.0) RETURN
      G(NOPENL)=(U(1)*RO(1)+DG(NOPENL))/2.*DT+G(NOPENL)
      G(NOPENR)=(-U(N1)*RO(N1)+DG(NOPENR))/2.*DT+G(NOPENR)
      DG(NOPENL)=U(1)*RO(1)
      DG(NOPENR)=-U(N1)*RO(N1)
C
      RETURN
      END
```

```
      SUBROUTINE STEP (AK,DX,DT,N1)
C     CALCULATES VALUES AFTER TIME STEP
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BEM(101), BEN(101), BES(101), X1(101), X2(101), X3(101)
      DIMENSION S1(101)
      COMMON /GEOM/ POS(6),THETA(4),AA,B,DL,KDPM
      COMMON /RATIO/ RATX(101),RATX1(101),RATX2(101),RAT(101)
      COMMON /C/ AM(101),AN(101),SIG(101),X(101)
      COMMON /C1/ AM1(101),AN1(101),SIG1(101)
      COMMON /C2/ AM2(101),AN2(101),SIG2(101)
      COMMON /C3/ P(101),A(101),AMAC(101),RO(101),T(101),S(101),U(101)
      COMMON /GAMMA/ GAM,GAM1,GAM2,GAM3,GAMC,GAM4,GAM5,GAM6
C
C     DETERMINE MAXIMUM ALLOWABLE TIME STEP
      N=N1-1
      BMAX=0.0
C
      DO 1 I=1,N1
      BES(I)=U(I)
      BEN(I)=A(I)+U(I)
      BEM(I)=-A(I)+U(I)
      IF (DABS(BEM(I)).GT.BMAX) BMAX=DABS(BEM(I))
      IF (DABS(BEN(I)).GT.BMAX) BMAX=DABS(BEN(I))
      S1(I)=SIG(I)
    1 CONTINUE
C
      DT=AK*DX/BMAX
C
C     EVALUATE COMPUTING VARIABLES TO BE USED FOR CHARACTERISTICS
      CALL SLOPE (BES,DT,X,X3,N1,1)
      CALL SLOPE (BEM,DT,X,X2,N1,1)
      CALL SLOPE (BEN,DT,X,X1,N1,1)
      CALL LININT (BEN,DX,X,X1,N1,RATX,RATX1)
      CALL LININT (BEN,DX,X,X1,N1,AM,AM1)
      CALL LININT (BEN,DX,X,X1,N1,AN,AN1)
      CALL LININT (BEN,DX,X,X1,N1,SIG,SIG1)
      CALL LININT (BEM,DX,X,X2,N1,RATX,RATX2)
      CALL LININT (BEM,DX,X,X2,N1,AM,AM2)
      CALL LININT (BEM,DX,X,X2,N1,AN,AN2)
      CALL LININT (BEM,DX,X,X2,N1,SIG,SIG2)
      CALL LININT (BES,DX,X,X3,N1,S1,SIG)
C
C     ESTABLISH A ROUGH GUESS FOR PRESSURE AND VELOCITY AT P
C
      DO 2 I=2,N
      AN(I)=(AN1(I)*SIG1(I)+AM1(I)+AN2(I)*SIG2(I)-AM2(I))/(SIG2(I)+SIG1(
     1I))
```

```
      AM(I)=(AN1(I)-AN2(I)+AM1(I)/SIG1(I)+AM2(I)/SIG2(I))/(1./SIG1(I)+1.
     1/SIG2(I))
    2 CONTINUE
C
C
C     ITERATE FOR PRESSURE AND VELOCITY AT P WHERE HPQ AND HPS REPRESENT
C     AVERAGE VALUES OF H ALONG CHARACTERISTICS
C
      DO 4 I=2,N
    3 ANT=AN(I)
      AMT=AM(I)
      EPS=0.
      HPQ=(AM(I)*SIG(I)*AN(I)*RATX(I)+AM1(I)*SIG1(I)*RATX1(I)*AN1(I))/2.
      HPS=(AM(I)*SIG(I)*AN(I)*RATX(I)+AM2(I)*SIG2(I)*RATX2(I)*AN2(I))/2.
      AM(I)=(AN1(I)-AN2(I)+AM1(I)/SIG1(I)+AM2(I)/SIG2(I)-HPQ*DT/SIG1(I)+
     1HPS*DT/SIG2(I))/(1./SIG1(I)+1./SIG2(I))
      AN(I)=(AN1(I)*SIG1(I)+AM1(I)+AN2(I)*SIG2(I)-AM2(I)-HPQ*DT-HPS*DT)/
     1(SIG1(I)+SIG2(I))
      EPS=(AM(I)+AN(I)-ANT-AMT)
      IF (EPS.GT..1) GO TO 3
    4 CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE SLOPE (BE,DT,X,X1,N1,NN)
C     CALCULATES CHARACTERISTIC PASSING THROUGH A GRID POINT
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BE(101), SL(101), X(101), X1(101)
C
C     DETERMINE IF END POINTS ARE TO BE EVALUATED
      L=1
      N=N1-1
      IF (BE(1).GT.0.0) L=2
      IF (BE(N1).GT.0.0) N=N1
      IF (NN.EQ.2) GO TO 2
C
C     COMPUTE AVERAGE SLOPE BETWEEN GRID POINTS
C
      DO 1 I=L,N
      K=I+1
      IF (BE(I).GT.0.0) K=I-1
      SL(I)=(BE(I)+BE(K))/2.
C
C     FIND X-INTERCEPT
    1 X1(I)=X(I)-SL(I)*DT
C
      RETURN
C
    2 DO 3 I=L,N
      SL(I)=BE(I)
    3 X1(I)=X(I)-SL(I)*DT
C
      RETURN
      END
```

```
      SUBROUTINE PORTX (N1,INDL,NOPENL,PHIL,PXL,PCL,TOL,INDR,NOPENR,PHIR
     1,PXR,POR,TOR)
C     CONTROLS CALCULATIONS AT BOUNDARY NODES
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /C3/ P(101),A(101),AMAC(101),RU(101),T(101),S(101),U(101)
      COMMON /STATUS/ NOVALV
      COMMON /CYC/ TOP(2),PXP(2),POP(2),IDATA(2),TMAX
      COMMON /GAMMA/ GAM,GAM1,GAM2,GAM3,GAMC,GAM4,GAM5,GAM6
C
C     IDATA=1 IF BOUNDARY STATIC PRESSURE SUPPLIED, IDATA=2 IF BOUNDARY
C     STAGNATION VALUE SUPPLIED
      PRL=(1.+GAM1*AMAC(1)**2)**(GAM/(GAM-1.))
      PRR=(1.+GAM1*AMAC(N1)**2)**(GAM/(GAM-1.))
C
      DO 6 I=1,2
      GO TO (1,2), I
    1 PR=PRL
      GO TO 3
    2 PR=PRR
    3 CONTINUE
C
C     DETERMINE STATIC OR STAGNATION PRESSURE AS APPROPRIATE
      K=IDATA(I)
      GO TO (4,5), K
    4 POP(I)=PXP(I)*PR
      GO TO 6
    5 PXP(I)=POP(I)/PR
    6 CONTINUE
C
C
C     LEFT HAND VALVE AND CORRESPONDING PRESSURE VALUES
      PHIL=0.D0
      NOPENL=2
      PXL=1.E-14
      POL=1.E-14
      TOL=1.E-14
      IF (NOVALV.EQ.1) GO TO 7
C
C     RIGHTHAND VALVE AND CORRESPONDING PRESSURE VALUES
      PHIR=0.D0
      NOPENR=2
      PXR=1.E-14
      POR=1.E-14
      TOR=1.E-14
      GO TO 8
    7 PHIR=1.D0
      NOPENR=1
```

```
      PXR=PXP(2)
      PCR=POP(2)
      TCR=TOP(2)
    8 CONTINUE
      CALL PORTS (PXL,1,INDL)
      CALL PORTS (PXR,N1,INDR)
      RETURN
      END
```

```
      SUBROUTINE PORTS (PXS,I,IND)
C
C     CALCULATES VARIABLES AT BOUNDARY NODES
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /STATUS/ NOVALV
      COMMON /C2/ AM2(101),AN2(101),SIG2(101)
      COMMON /C1/ AM1(101),AN1(101),SIG1(101)
      COMMON /C/ AM(101),AN(101),SIG(101),X(101)
      COMMON /GAMMA/ GAM,GAM1,GAM2,GAM3,GAMC,GAM4,GAM5,GAM6
C
C     DETERMINE HAND OF PORT
C      LEFT HAND PORT
      IF (I.EQ.1) IHAND=1
C      RIGHT HAND PORT
      IF (I.NE.1) IHAND=2
      GO TO (1,2), IHAND
C     CLOSED PORT
    1 AM(I)=0.D0
      IND=1
      AN(I)=AN2(I)-AM2(I)/SIG2(I)
      RETURN
    2 IF (NOVALV.EQ.1) GO TO 3
      AM(I)=0.D0
      IND=1
      AN(I)=AN1(I)+AM1(I)/SIG1(I)
      RETURN
    3 A=AN(I)*SIG(I)
      U=AM(I)/GAM1
      IF (U.GT.A) GO TO 4
C     SUBCRITICAL OUTFLOW
      AN(I)=PXS**GAM6
      AM(I)=AN1(I)*SIG1(I)+AM1(I)-AN(I)*SIG(I)
      IND=9
      RETURN
C     SUPERCRITICAL OUTFLOW
    4 AN(I)=(AN1(I)*SIG1(I)+AM1(I)+AN2(I)*SIG2(I)-AM2(I))/(SIG2(I)+SIG1(
     1I))
      AM(I)=(AN1(I)-AN2(I)+AM1(I)/SIG1(I)+AM2(I)/SIG2(I))/(1./SIG1(I)+1.
     1/SIG2(I))
      IND=8
      RETURN
      END
```

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